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ABSTRACT

This paper consists of an address given at the Conference of Canadian Mathematicians and Mathematics Educators on June 4, 1978. The author sets the stage for concluding remarks with examples, stories, illustrated anecdotes, and observations of the influence of several "mathematicians" and "mathematics interpreters" on curriculum development. He concludes that mathematicians do have much to offer curriculum development and proceeds to list relevant strong points. The address ends with a discussion of some trouble spots that need consideration in maintaining a working relationship between mathematicians and curriculum developers. (MN)

THE MATHEMATICIAN's CONTRIBUTION TO CURRICULUM DEVELOPMENT*

Gerald R. Rising
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Nine times out of ten, I have found, when someone else chooses a topic for you to address, you and your prospective audience are in trouble. As only one of a number of examples of how this statistical law applies to me, I offer my supposed expertise in, and recent string of papers on, basic skills in mathematics. As a matter of fact, Steve Brown and I are even now trying to find a way around the title "Evaluation of Basic Skills" for an upcoming NCTM yearbook. Indeed it is this law that is one of several reasons for my decision to withdraw from the lists. This talk is my last for some time.

But my assigned subject here happily meets the ten per cent criterion. I feel that I am exactly the person who should speak to this topic. That doesn't necessarily protect you. You may still be in trouble; however, you will know exactly where to place the blame when I don't bring it off: not on the topic assignment, on me.

Be sure that you understand why I feel that I should speak to this subject. I do not speak as a mathematician but as a reasonably astute observer of many of the mathematicians who have made or who have attempted to make contributions to curriculum development in school and college mathematics over the past 25 years. My mathematics credentials are technically reasonable - I hold a master's degree in mathematics from a major U. S. university - but basically deficient. For me to pass before you as a mathematician, as I use the term here, would be to dissemble; I would by so doing place myself in company with the majority of so-called college and university mathematicians in my country. By mathematician here I mean mathematical scholar, that is a person contributing in a major way

*Invited address, Queens University, Kingston, Ontario, Canada, June 4, 1978, Conference of Canadian Mathematicians and Mathematics Educators.

to new knowledge in this field. To that I and most others are outsiders looking in.

In at whom? The following people, all of whom I have known reasonably well, characterize this group of mathematicians who have contributed to curriculum development for me: Peter Hilton, Paul Kelly, Morris Kline, (breaking alphabetic order to include two statisticians together) Bill Kruskal and Fred Mosteller, E. E. Moise, George Papy, Dan Pedoe, Paul Rosenbloom, Arnold Ross, Albert Tucker, and Herbert Vaughan. Please do not read into that list more than I have indicated. Some of these mathematicians I know as personal friends; with others my association is that of a student; Moise, in particular, I know largely through his former student and colleague and my current colleague, Steve Brown.

Having segregated the mathematicians and while I am name dropping, I add a second list, a list of mathematics interpreters, people who in one way or another have made significant positive contributions to mathematics curriculum or to our thinking about mathematics curriculum but who do not belong on the first list. This should be a much longer list. That it is not is at once a reflection on our mathematics education community and on contemporary politics. I will not refer to them all in the sequel, but I am prepared to defend the inclusion of each one on my list: Max Beberman, Ed Begle, Peter Braunfeld, Bob Davis, Zed Dienes, Arthur Engel, Bob Exner, Vince Haag, Harold Jacobs, Burt Kaufman, David Page, Frederique Papy, Warwick Sawyer, and Bob Wirtz.

In a very real way this second group helps to define and further restrict the first. Clearly this second list includes people who enjoy senior positions in mathematics departments of major universities and also includes several people who would have difficulty making an initial screening of candidates for a local community college. Now, having scandalized practically everyone, let me say that I will easily slip from my own categorizations and that these arbitrary distinctions should not strongly influence your own thinking about curriculum development. Still they will serve.

The remainder of this paper will be in two parts. In the first I will tell some stories, or as my more formal co-workers would say, provide anecdotal evidence. This will, I hope, serve as more than light entertainment. The examples should provide evidence to support the statements of part two. In that second part I will attempt to make some sense out of the chaos of part one, I will try to systematize my observations in the form of conclusions, and perhaps most important I will try to set an agenda for the full and open discussion that I hope will follow this paper.

Some Experiences in the Land of the Curriculum Developers

My first brush with a textbook writer is worth recounting. It occurred at my first NCTM meeting in Boston in 1950 or 1951. One evening I got on the hotel elevator and found myself face to face with an almost apoplectic Walter Hart. Now Water Hart you should understand, was one of our most prolific school mathematics textbook writers. Together with his brother Bill, who I am proud to say was later my officemate at the University of Minnesota and who wrote college texts, they probably wrote more books than Trollope's mother. And popular. A couple of years ago I saw an advertizement for one of Bill's calculus books claiming over a million copies sold. His royalties on that book were over a dollar a copy. But back to the story: I asked Walter if he was ill. "You're damned right I'm ill," he said. "I've just had a conference with my publishers. They surveyed classroom teachers for suggestions about my trig text and the one that they think is most important is four digit entry log tables to avoid interpolation." (I cannot resist noting that that story should suggest some serious problems associated with the PRISM project which surveys teachers for recommendations for change.)

My contact with New Math curriculum development was largely indirect. I was in Connecticut when SMSG was at Yale and knew many of the participants. I reviewed the Commission on Mathematics recommendations and I participated in the evaluation of the SMSG, UICSM and Ball State programs when I was mathematics director at the Minnesota National Laboratory. Of course we also used some of their materials in schools with which I was associated. Much more important and I believe central to my own learning about curriculum development, I worked rather closely with Paul Rosenbloom, a truly remarkable man.

Now I could take up the rest of this paper easily with stories about this gifted and creative mathematician and teacher, each of which would be to the point here. Instead I'll just talk about him and save most of the stories for another time.

Paul Rosenbloom represents to me at once much of what was right and much of what was wrong with New Math. In fact almost everything about Paul was, when I knew him, both right and wrong. He was a great leader, yet a poor administrator. He was a poor speaker yet a great teacher. He was a great conceptualizer, yet his record of seeing his ideas through to completion was poor. Those characteristics came together in a way that deeply impressed many of us but that made him an inconsequential dabbler to others.

Early in our not always comfortable association - I was his assistant director at the MINNEMATH Center of the University of Minnesota - Paul told me something that I suggest is reasonable advice to each of you: Develop a nice lesson or two suitable to each grade level, he said, because you'll often be called upon to teach when you visit schools. At the same time I cannot believe he followed his own advice. I saw him teach quite a few lessons to everyone from nursery school children to mathematics graduate students, but never the same lesson twice. What I saw convinces me that he has forgotten more genuinely creative, charming lessons than the rest of us have developed.

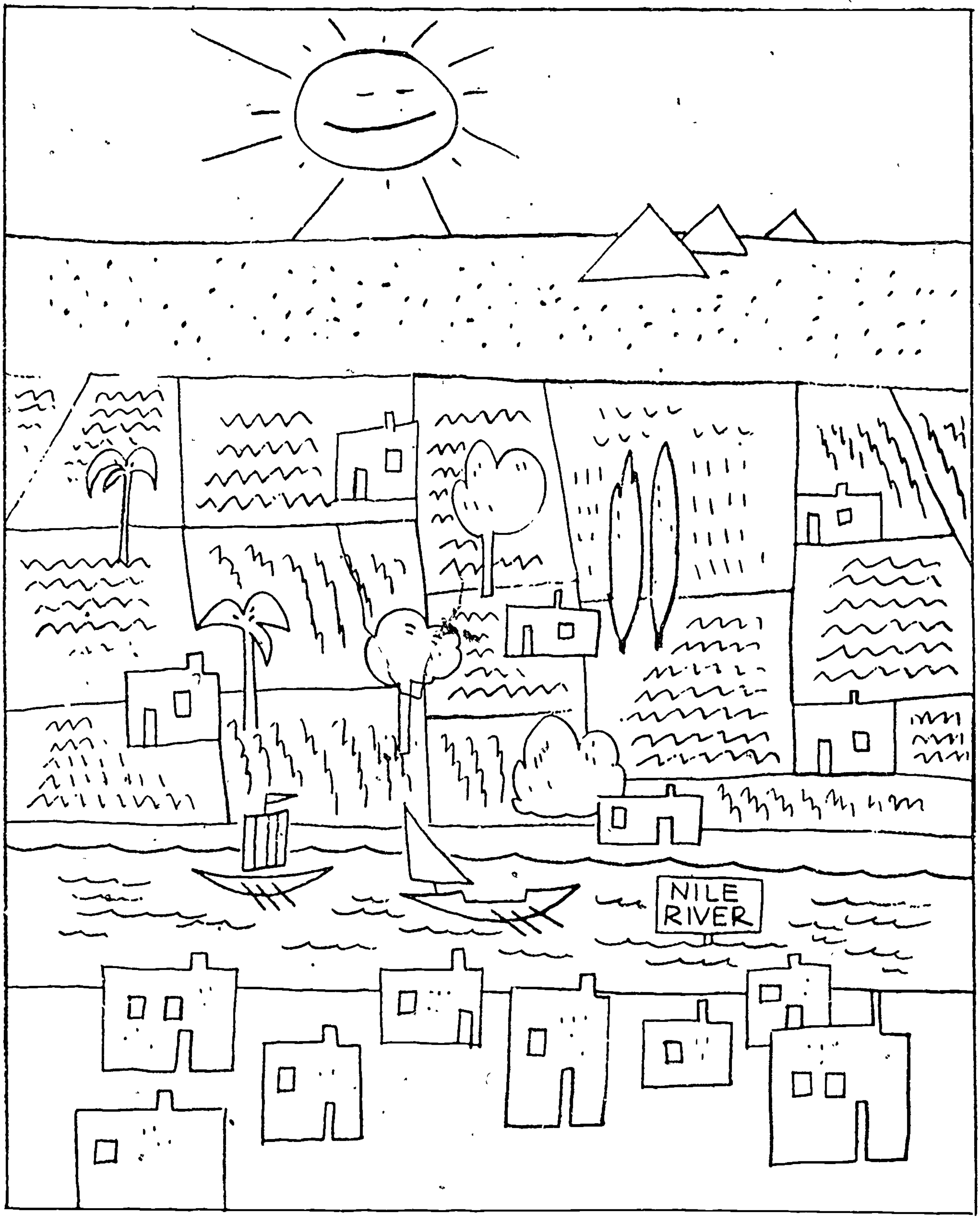
When I was with Paul, activities at Minnesota were coming unhinged. The delightful, rich stories that he created with their splendid characters, Ugboo, Tal and Ahmes, were being edited out of whack; his games, most notably Mathematical Golf, were being eliminated; his conceptual papers were lost in the shuffle; the science side of the MINNEMAST Project was reducing the math program to computation drill; and the critics were beginning to outnumber the supporters.

And so you see in microcosm the story of New Math: Some rich creative ideas - and, by the way, even some trash at the source - reduced by that meat-grinder of second-stringers, second guessers and commercializers into the same kind of pablum we see regularly served to children in [redacted] Ed Moise took a phrase familiar to us in the States describing the Battle of Lexington starting our Revolution, "the shot heard 'round the world," and reworked it slightly to describe New Math curriculum revision as "the shot heard 'round the immediate vicinity". When I worked with Paul Rosenbloom even that vicinity was shrinking.

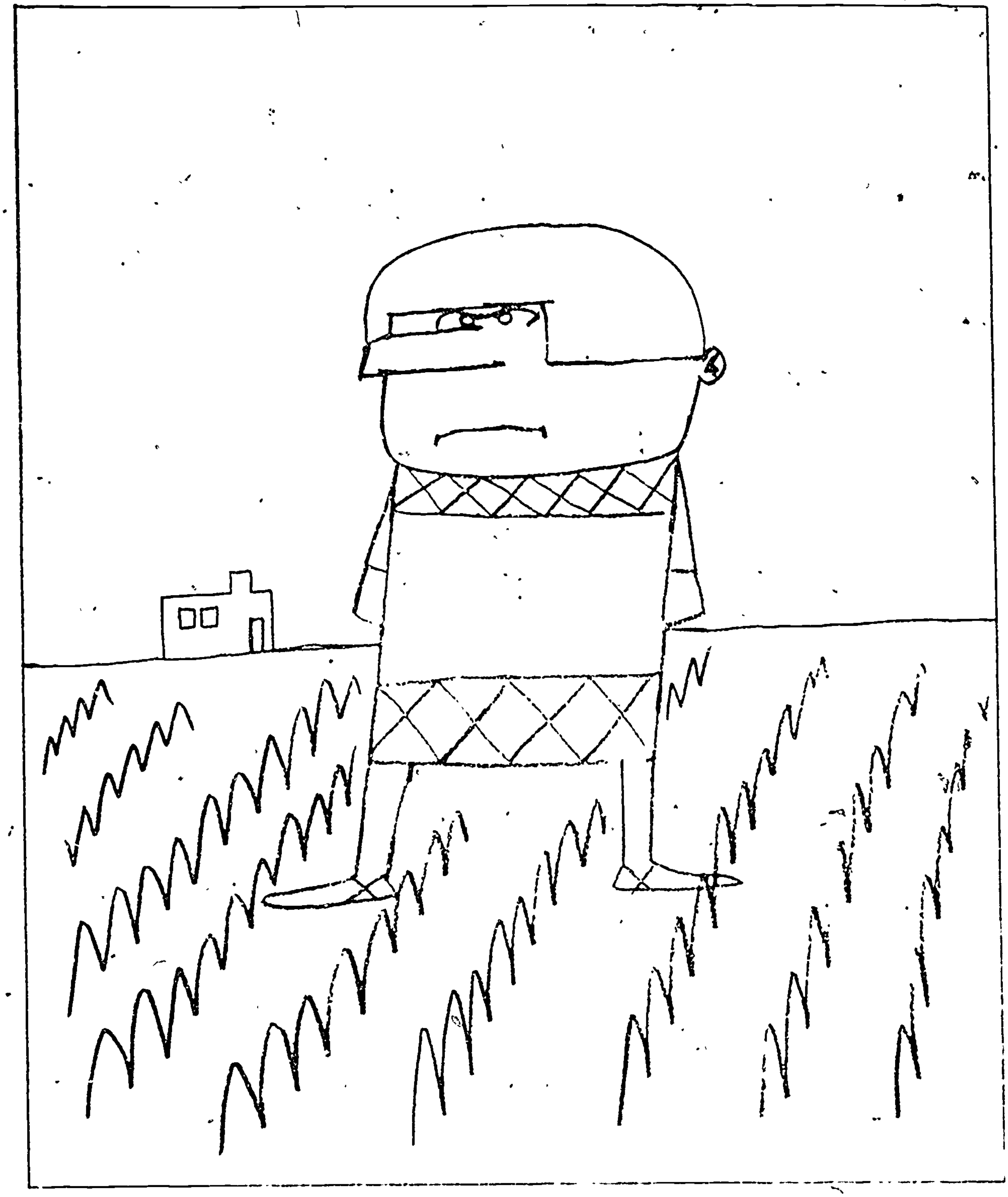
Paul has severely reduced his activities in mathematics education. This was by his own choice for reasons many of which are unrelated to this subject. But for whatever the reason, our loss is great.

I will try to give you some of the flavor of Paul's lively wit through the story of Ahmes the Taxpayer. Like so many Egyptian records (some burned in Alexandria) just so this delightful story with its associated deep conceptual basis is all but lost to us today.

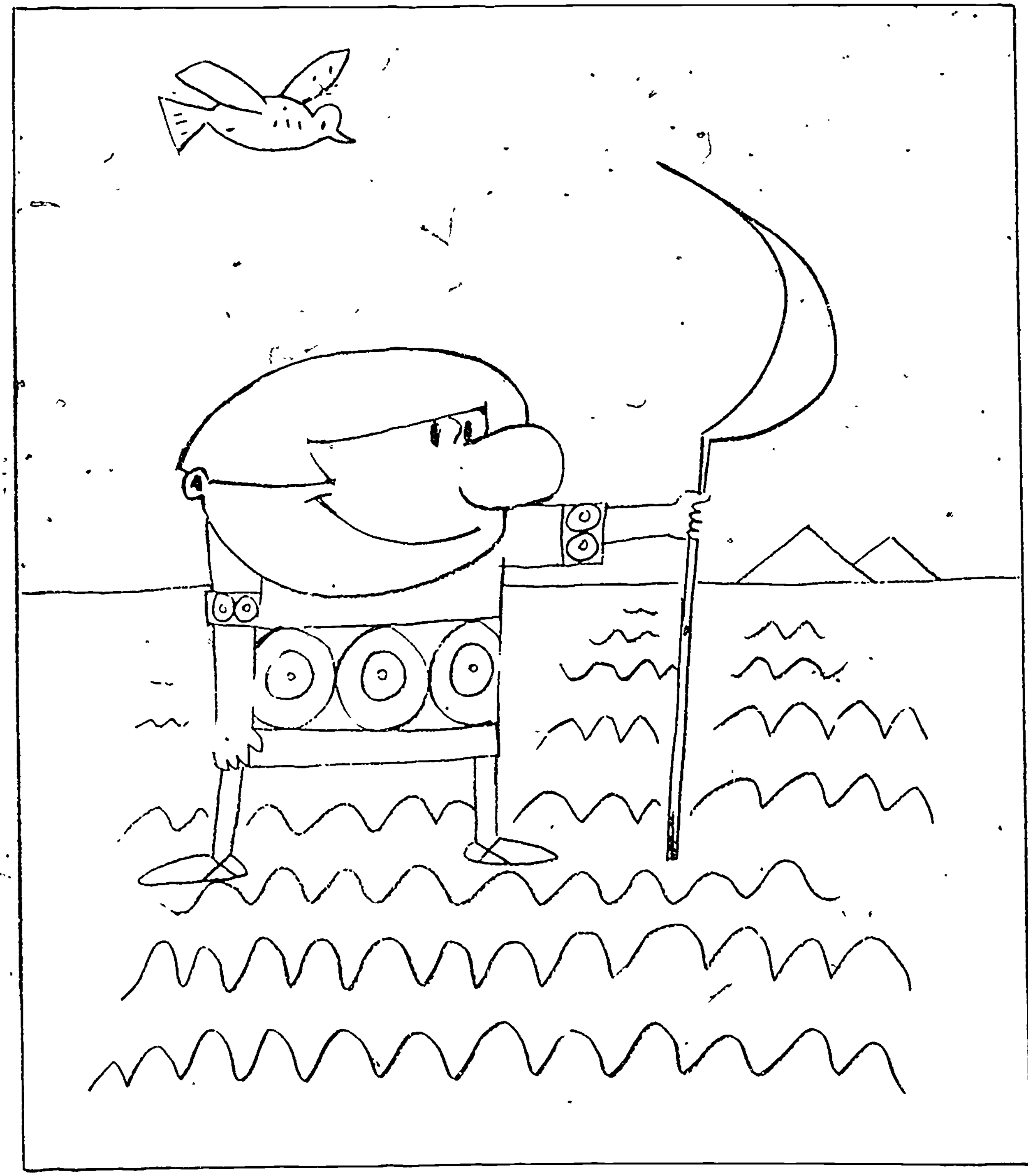
"Ahmes and the Tax Collector"



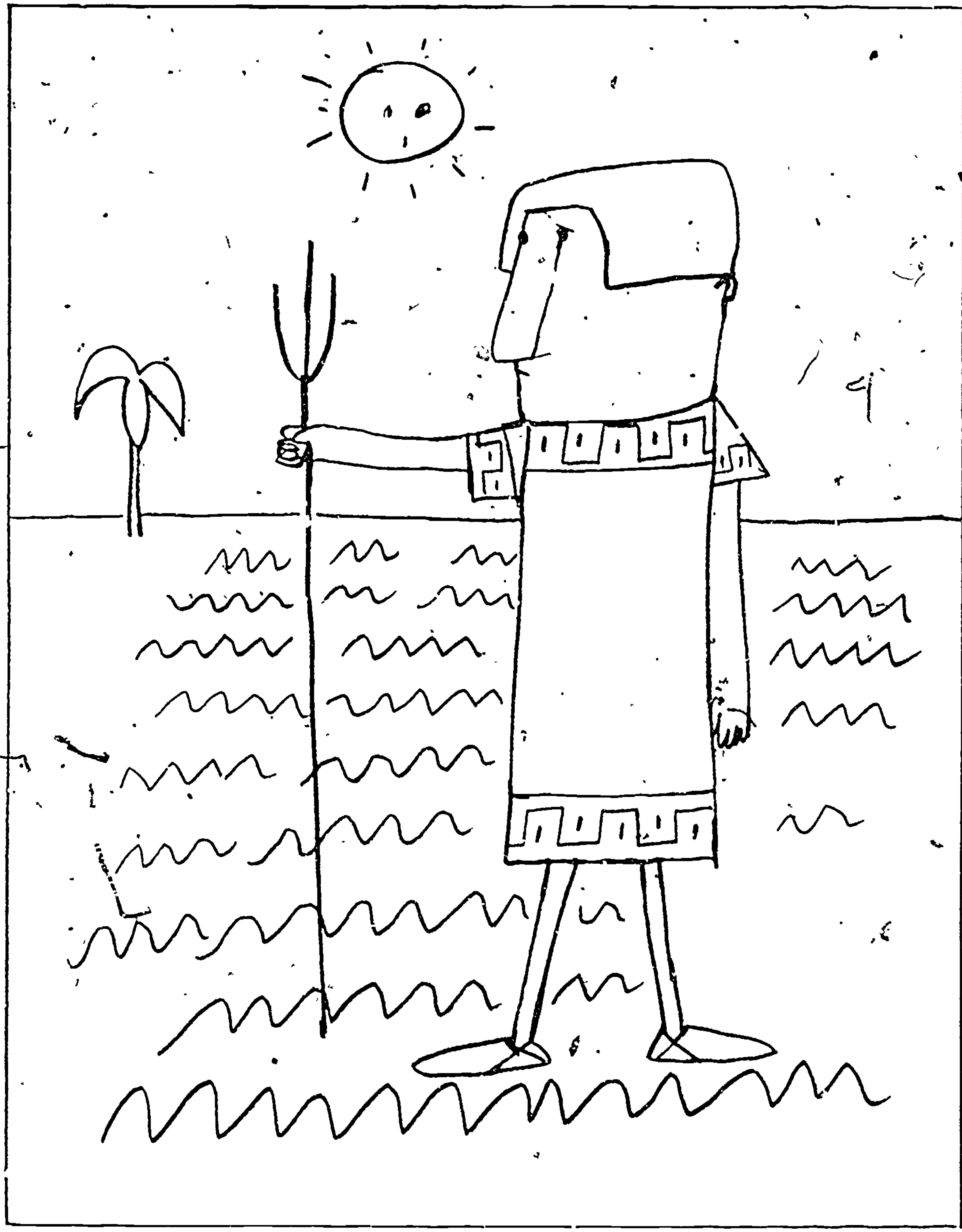
This is a country. It is called Egypt.



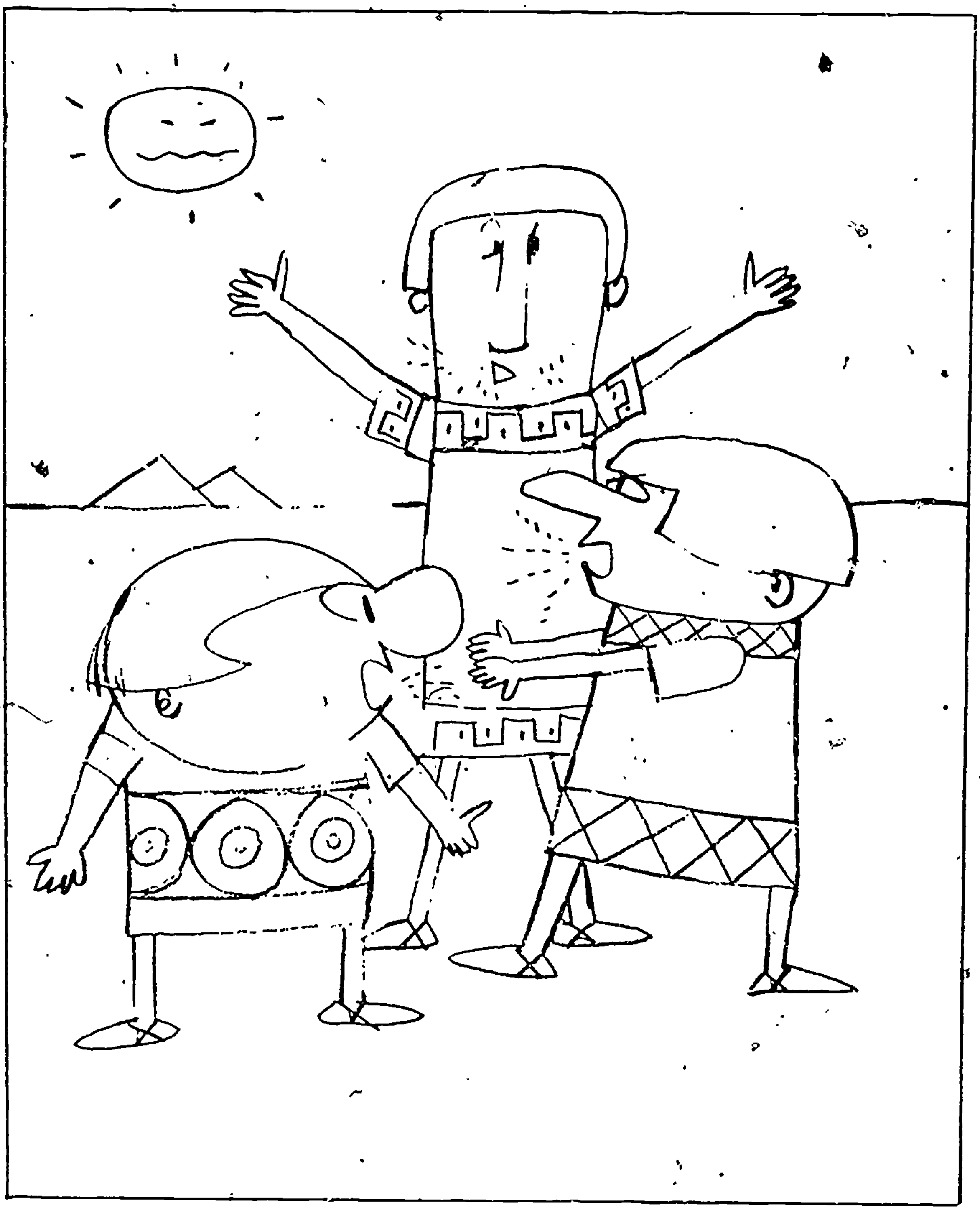
This is Ahmes, an Egyptian farmer. He is standing in his field.



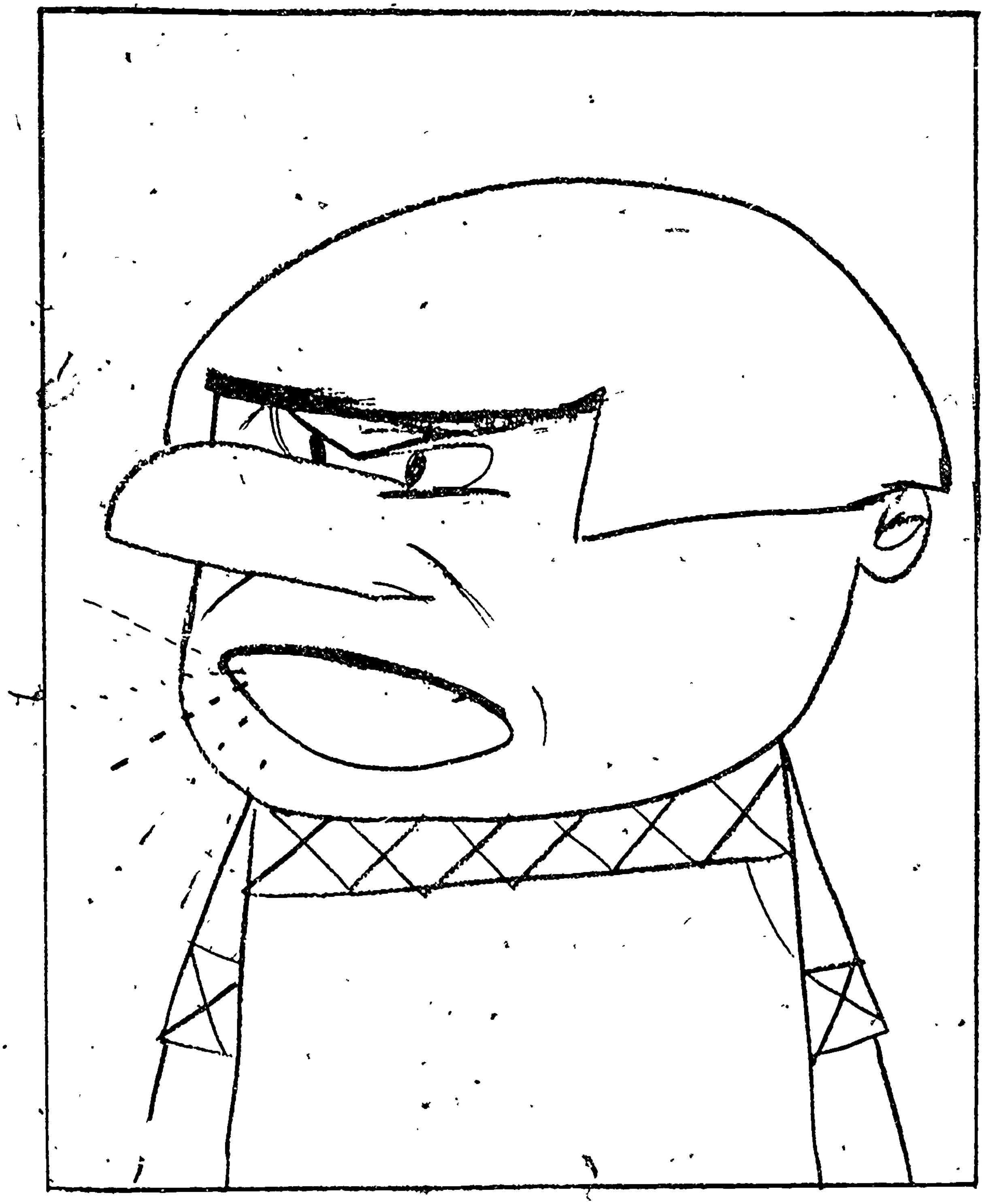
This is another farmer who is Ahmes' neighbor. He is standing in his field. His name is Tehuti.



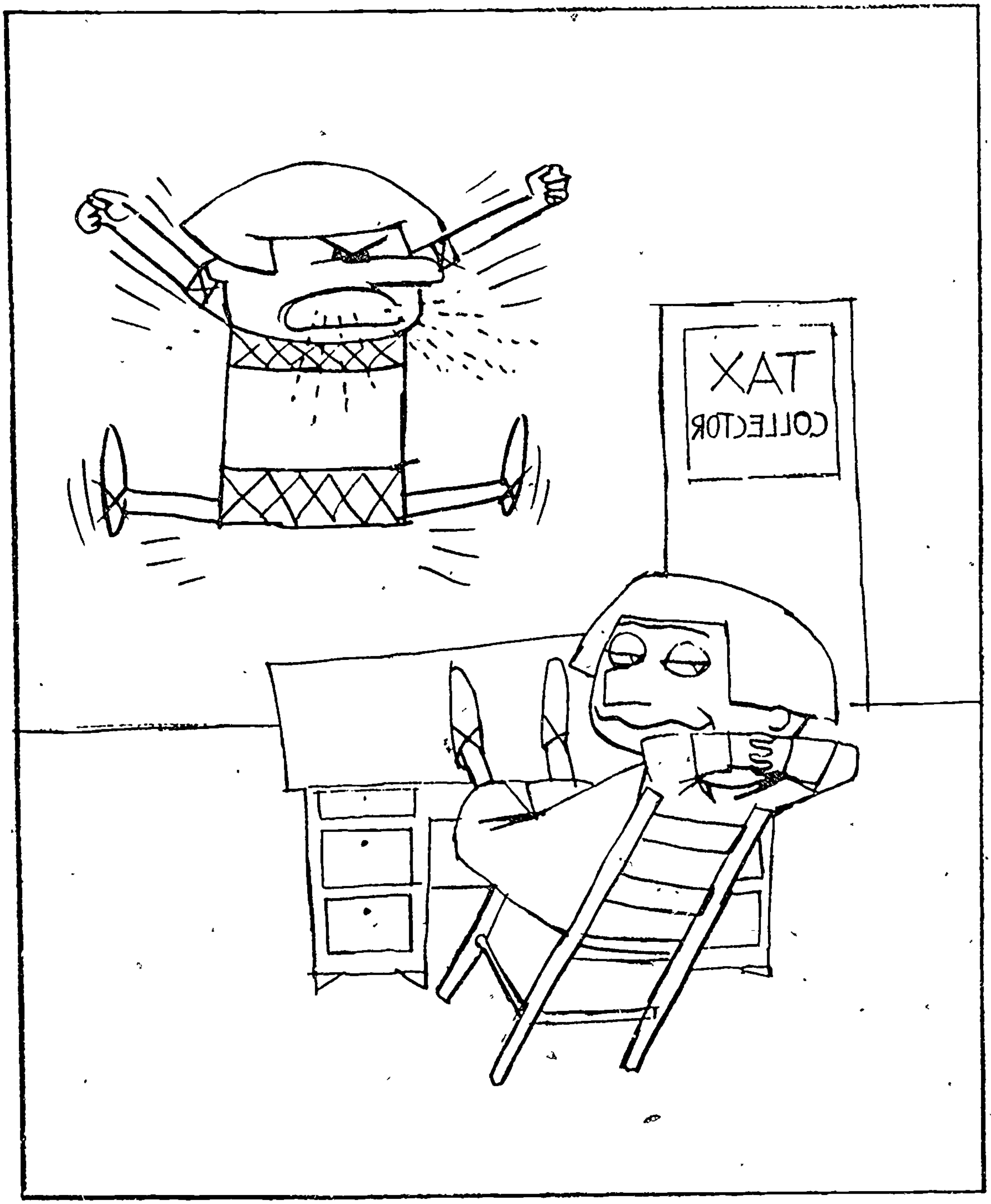
This is a third farmer who is another neighbor. He is standing in his field. His name is Tepi.



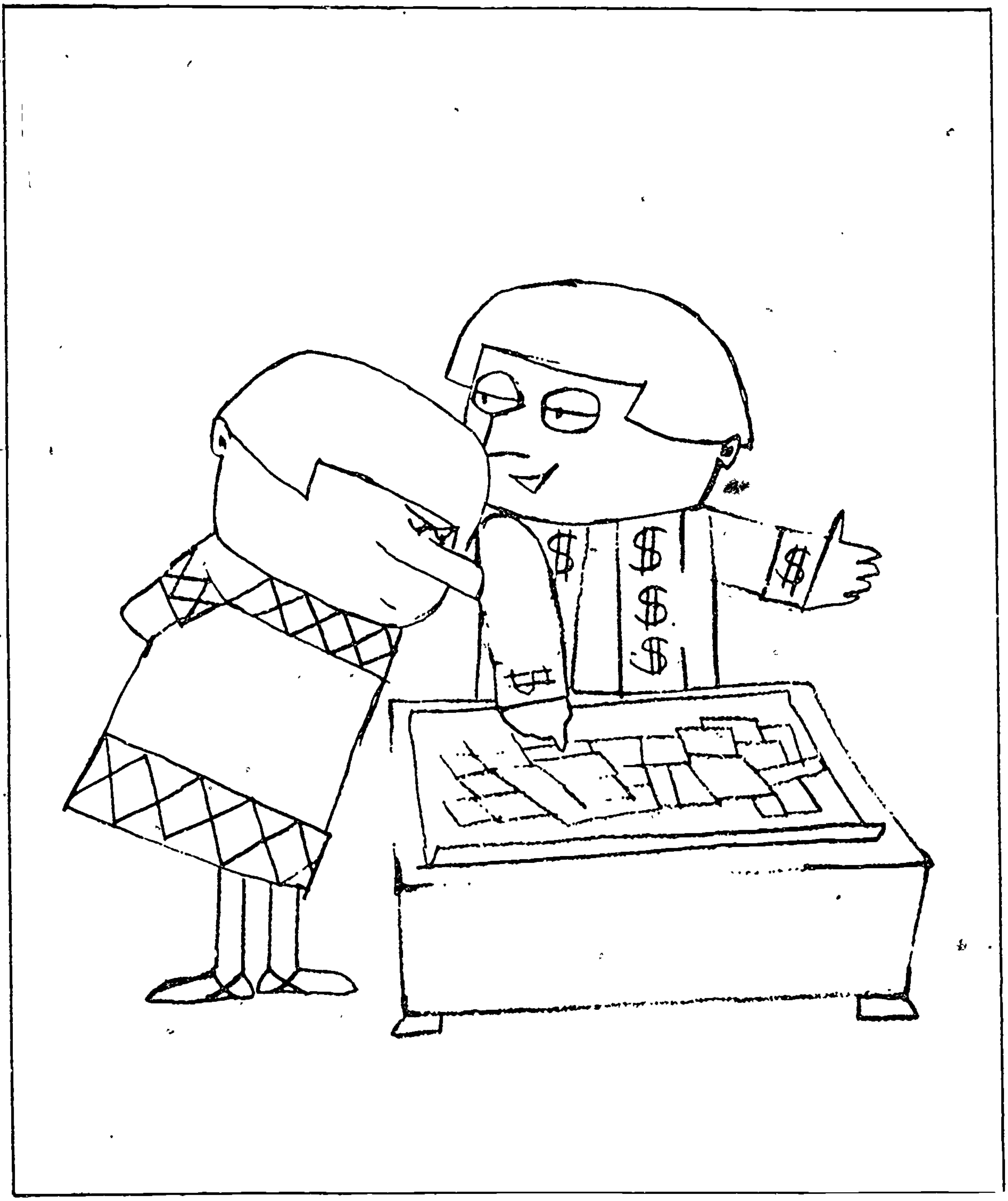
One day the three farmers were talking about the taxes they had to pay.



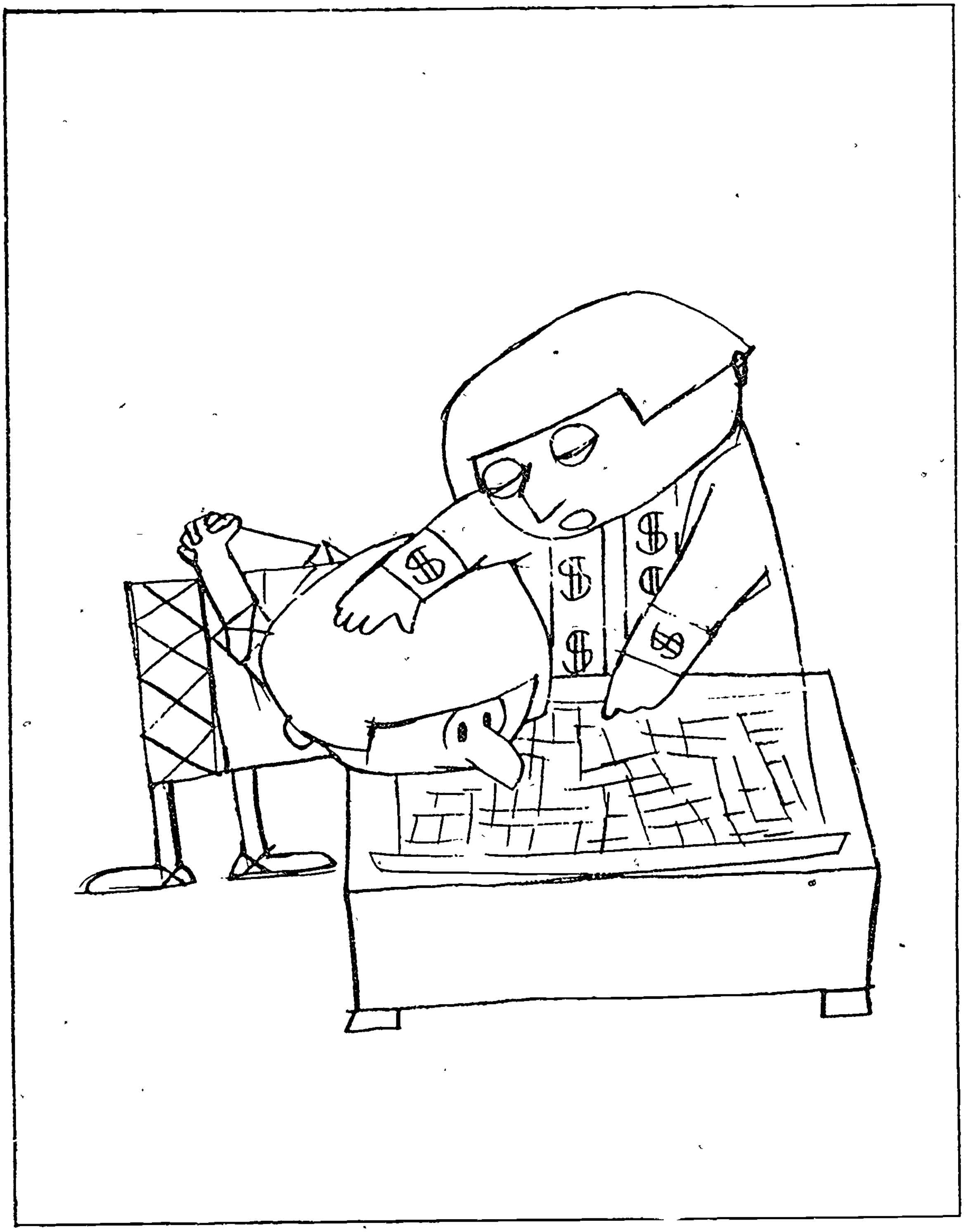
Ahmes said, "I don't know why I should have to pay more taxes than you do. It's not fair."



He went to the tax collector. The tax collector could tell that Ahmes was unhappy.

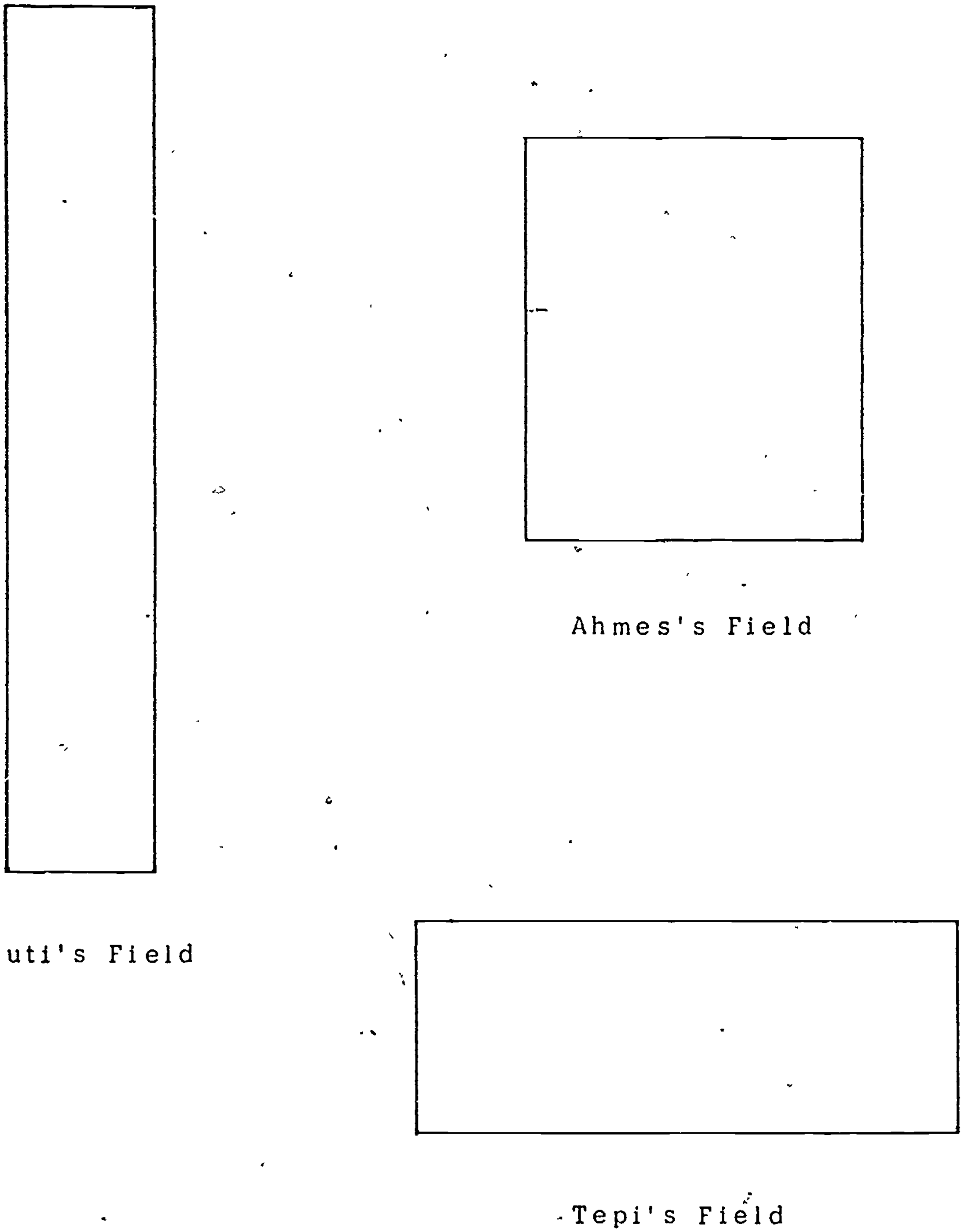


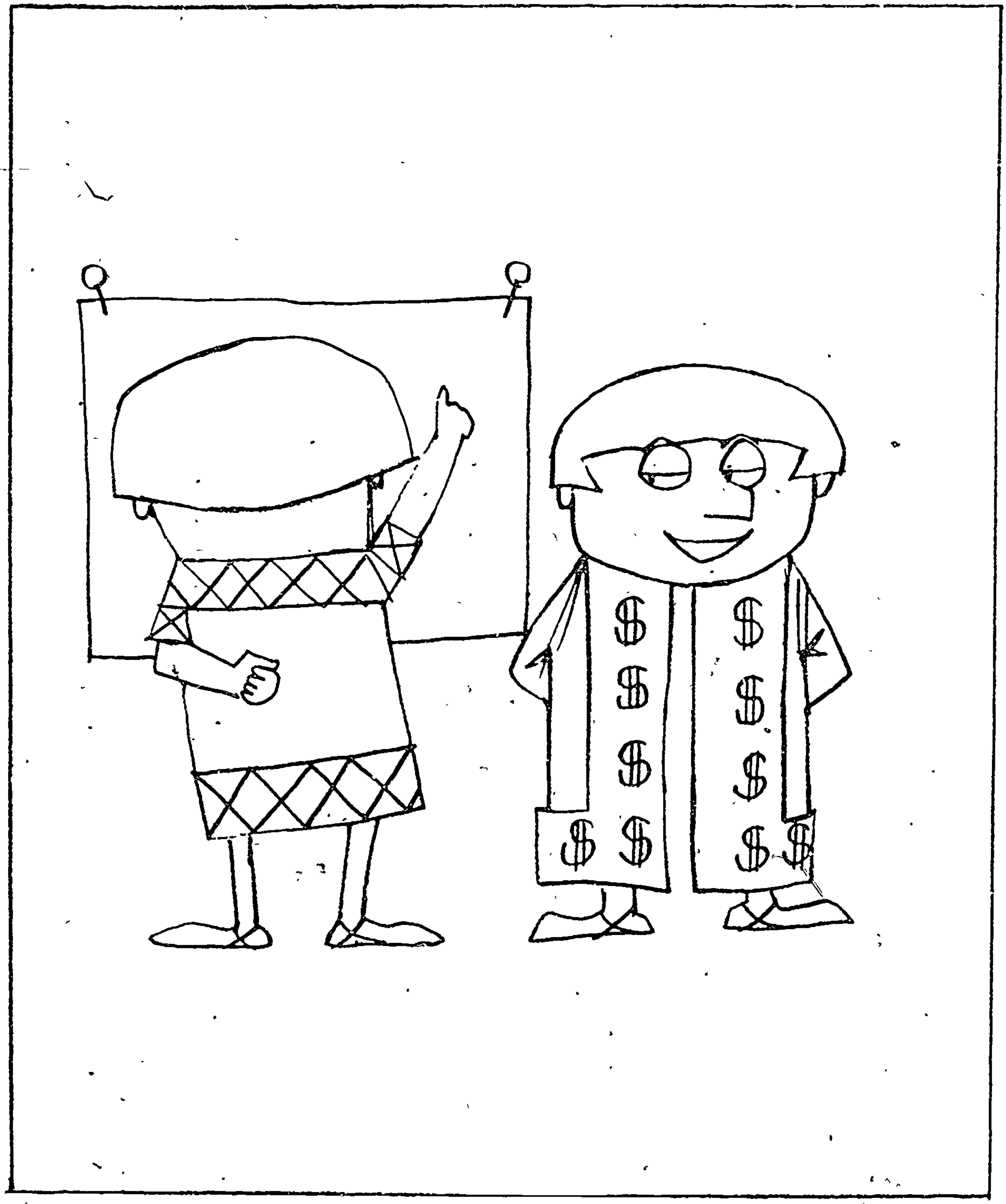
The tax collector took a big map out of his drawer. He said, "Ahmes, on this map is a picture of every farmer's field in our part of Egypt."



"Here is the picture of your field, and here are the fields of Tehuti and Tepi."

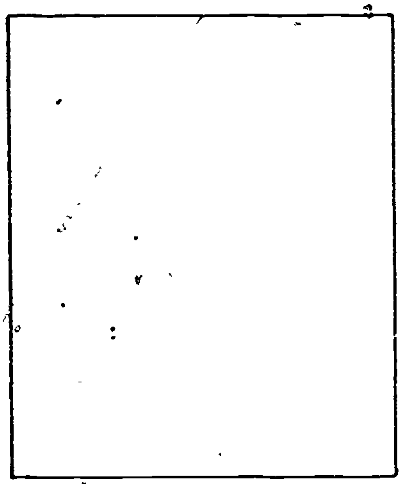
AHMES AND THE TAX COLLECTOR



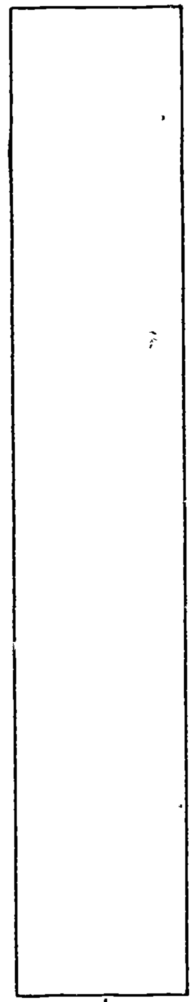


"Ahmes, it is true that Tehuit's field is longer than yours, but yours is wider. Tepi's field is wider than yours but yours is longer."

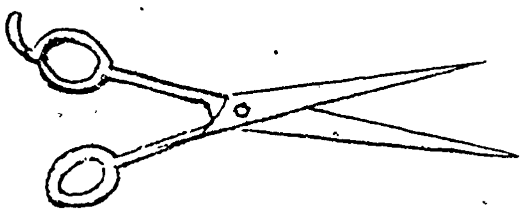
AHMES AND THE TAX COLLECTOR



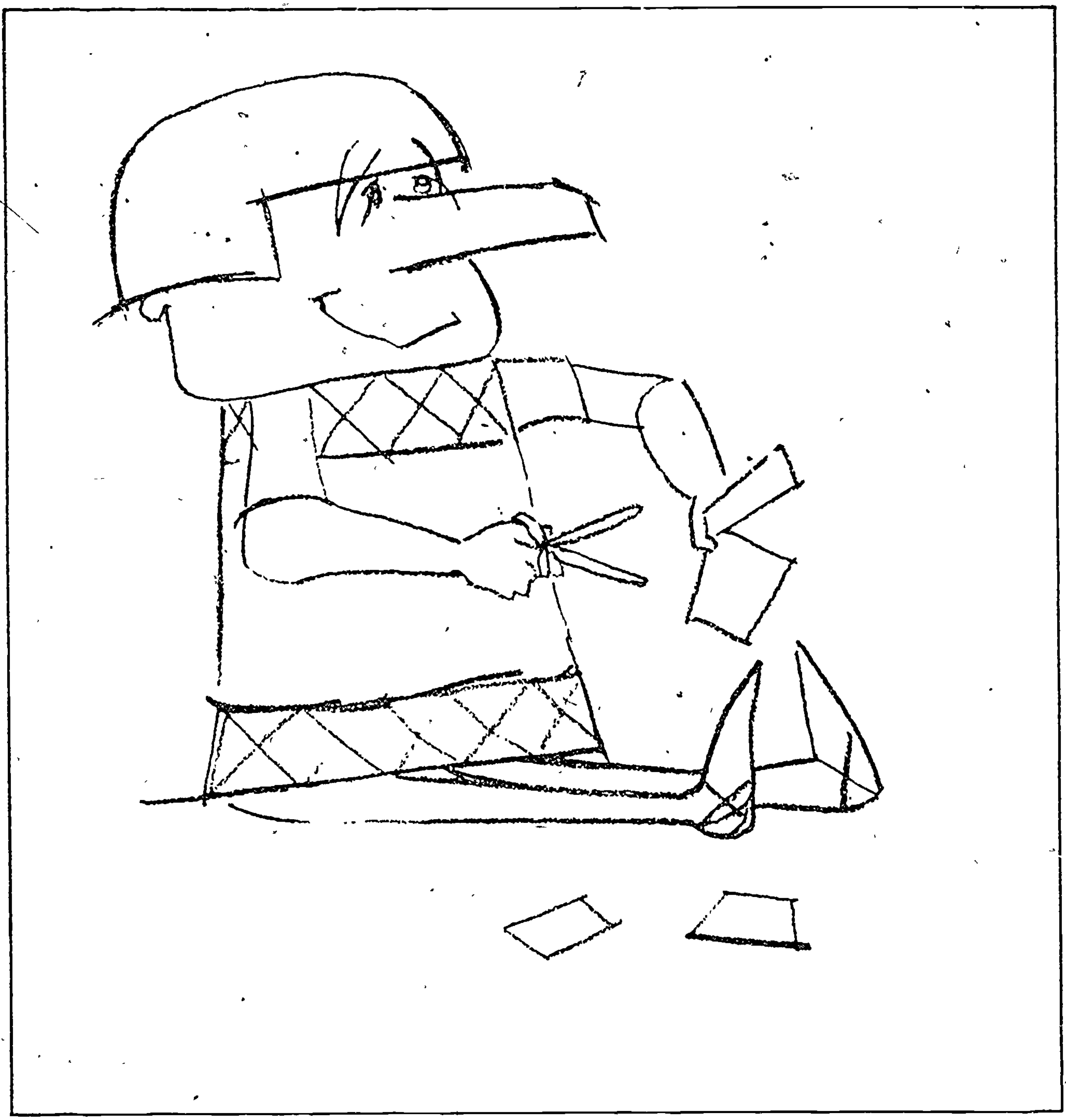
'Ahmes' Field



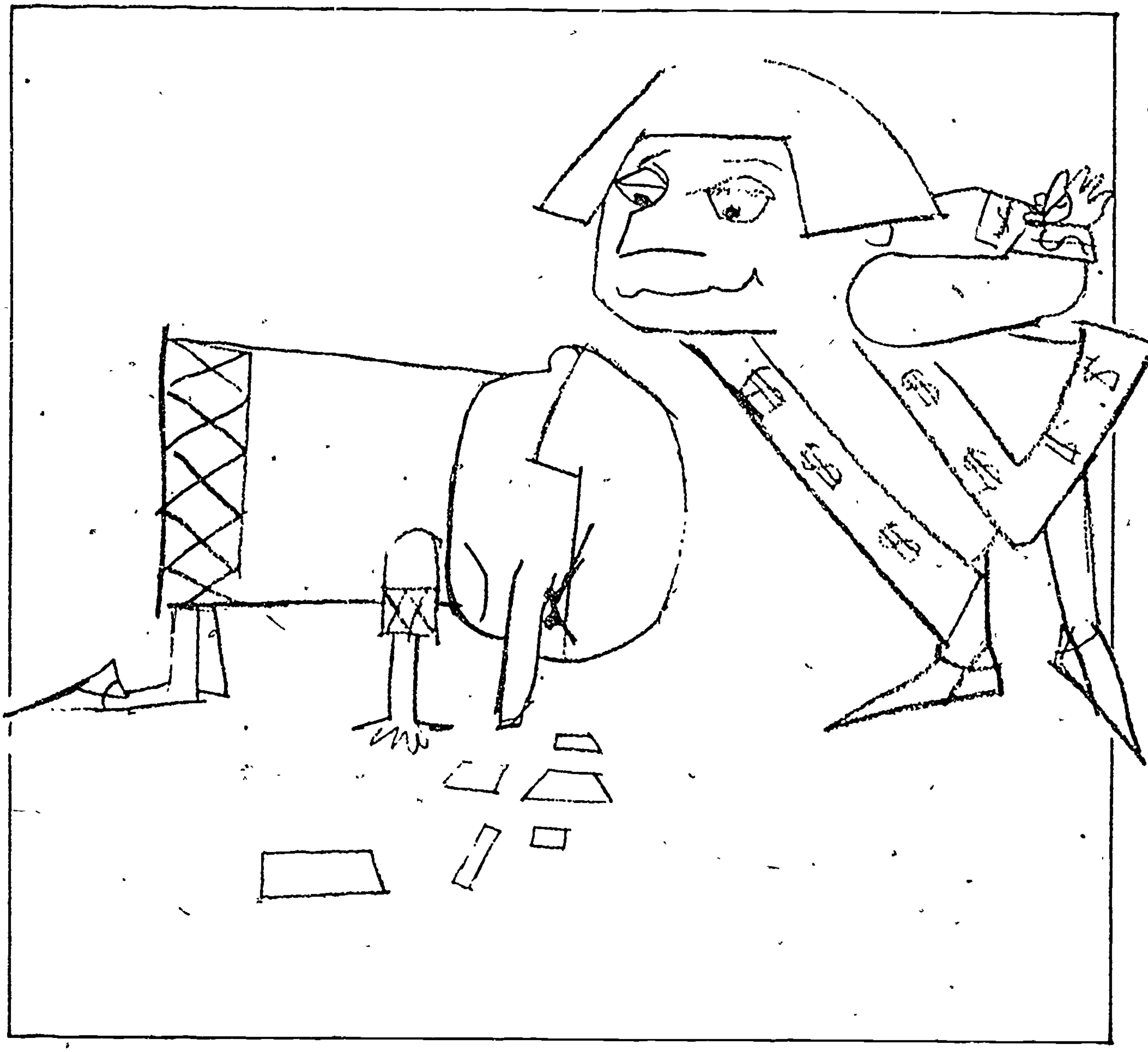
Tehuti's Field



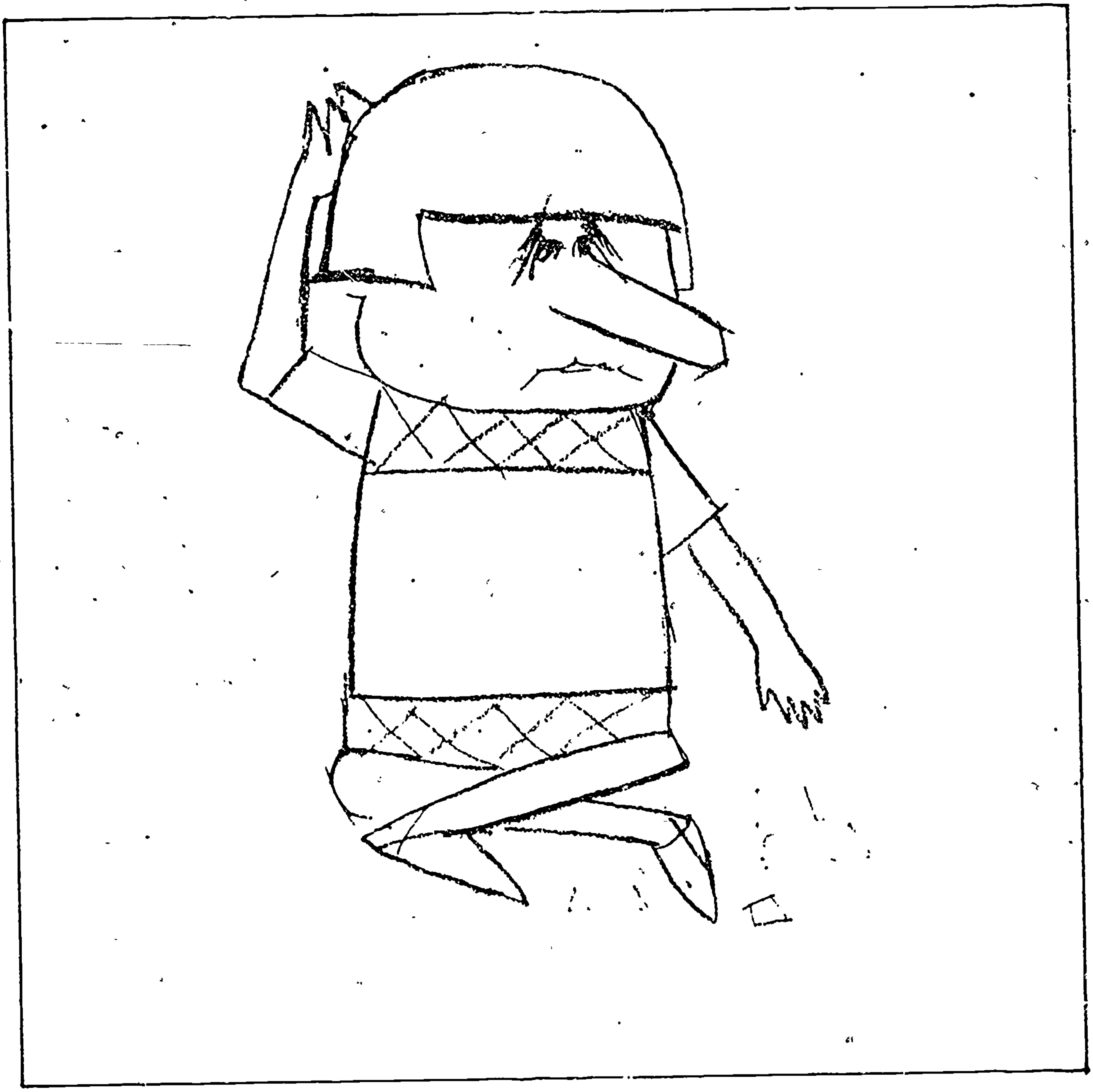
The tax collector gave Ahmes a picture of his field, a picture of Tehuti's field, and a pair of scissors.



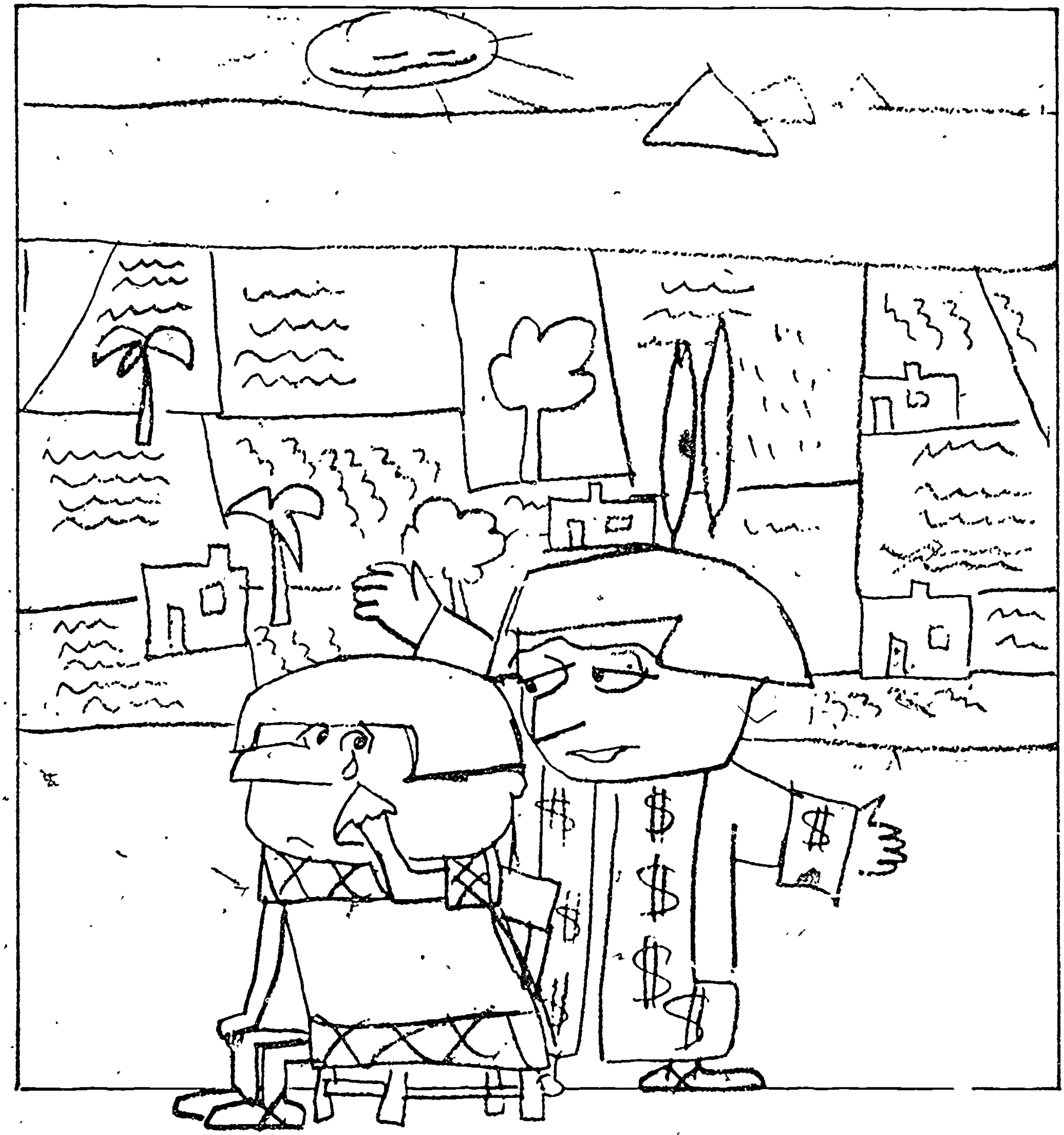
"Ahmes, if you cut Tehuti's field with the scissors and fit the rectangular pieces on your field you will see that your field is bigger than Tehuti's field."



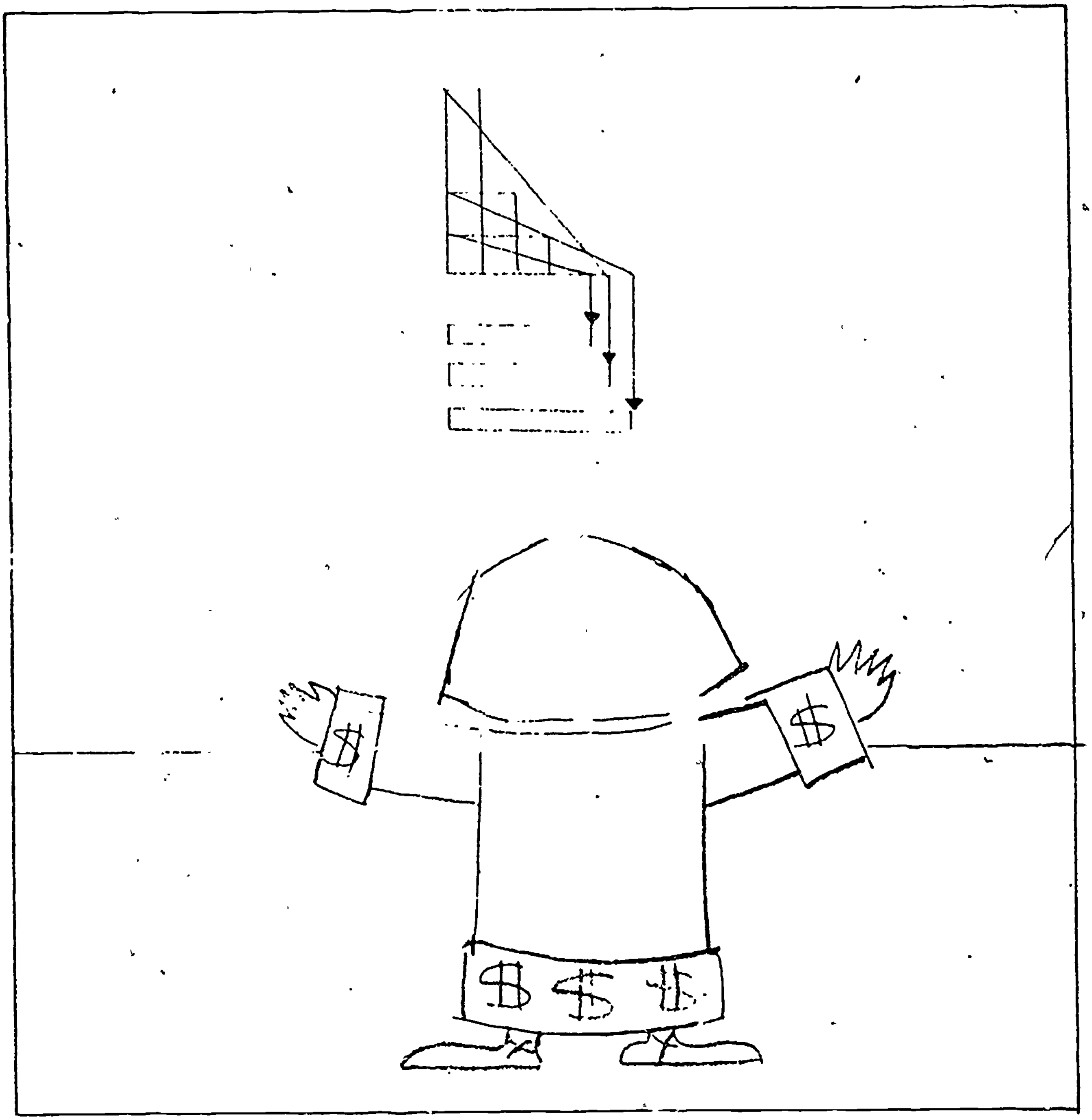
"Here you have a picture of Tepi's field. If you cut his field you will notice that yours is larger. By the way, Ahmes, whose field is larger, Tepi's or Tehuti's?"



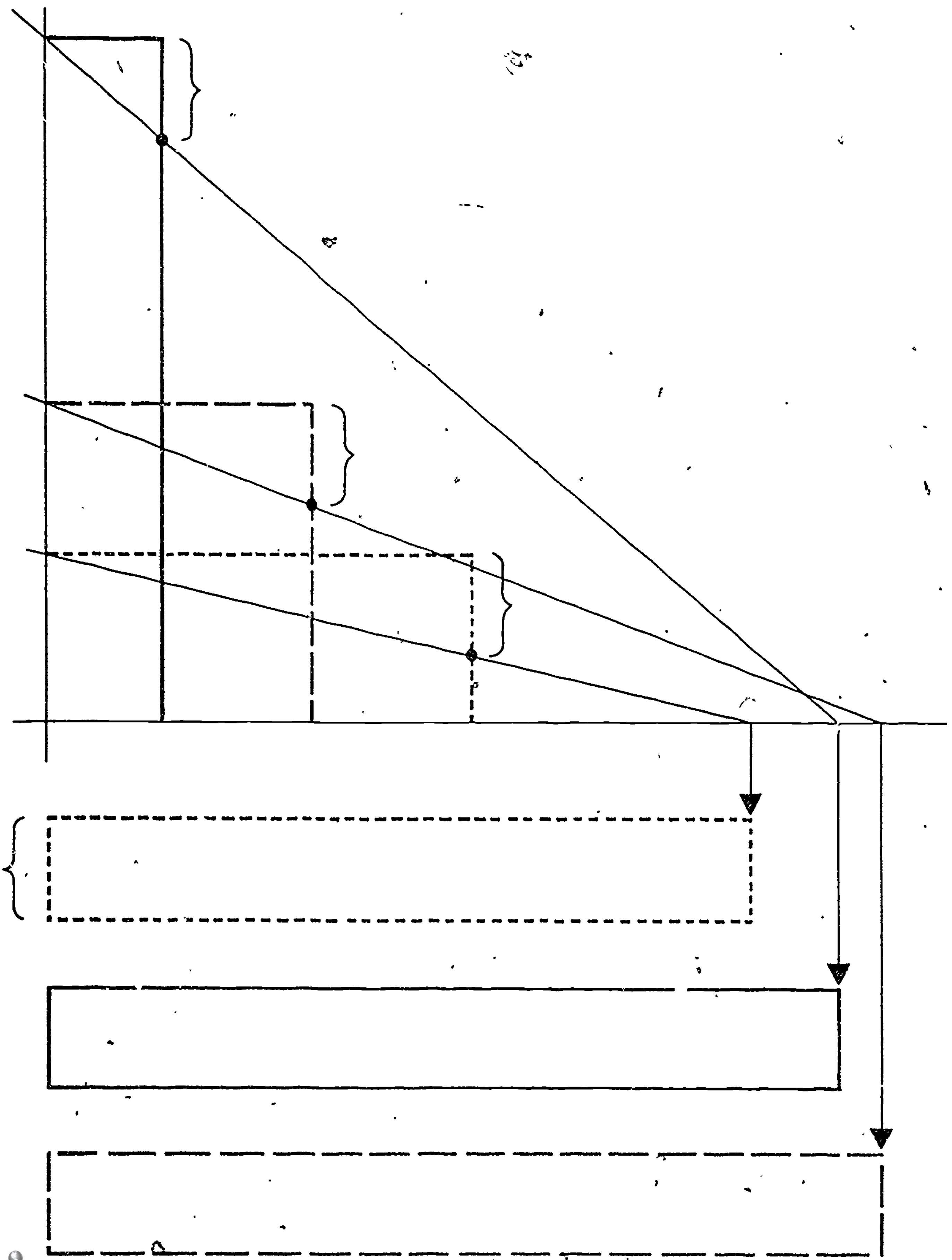
Because Ahmes had cut the pictures of Tepi's field and Tehuti's field he had a hard time finding that Tepi's field was smaller than Tehuti's.

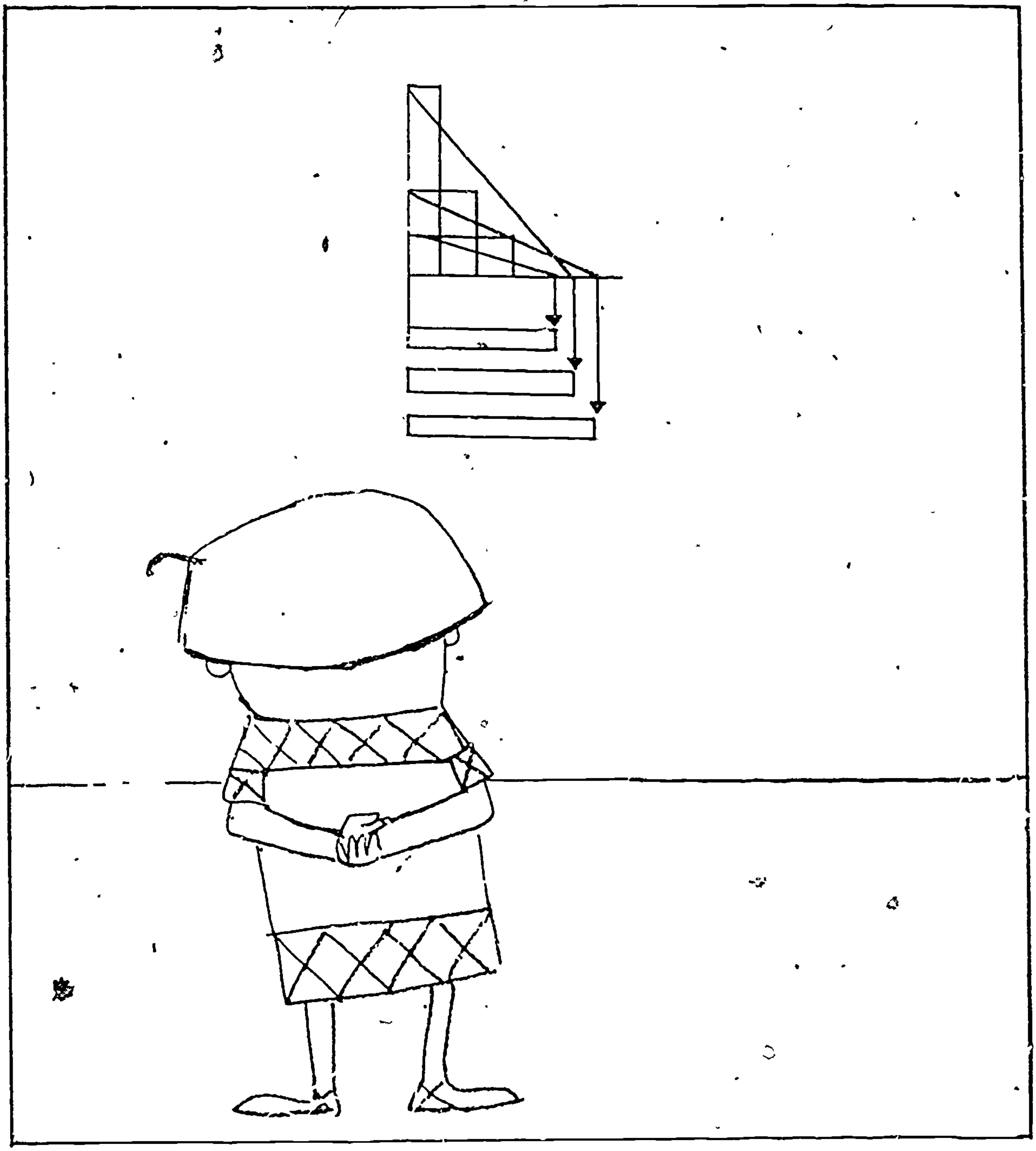


"Now Ahmes you know the problem I have finding how much tax each farmer of Egypt must pay, and Egypt has many farms."

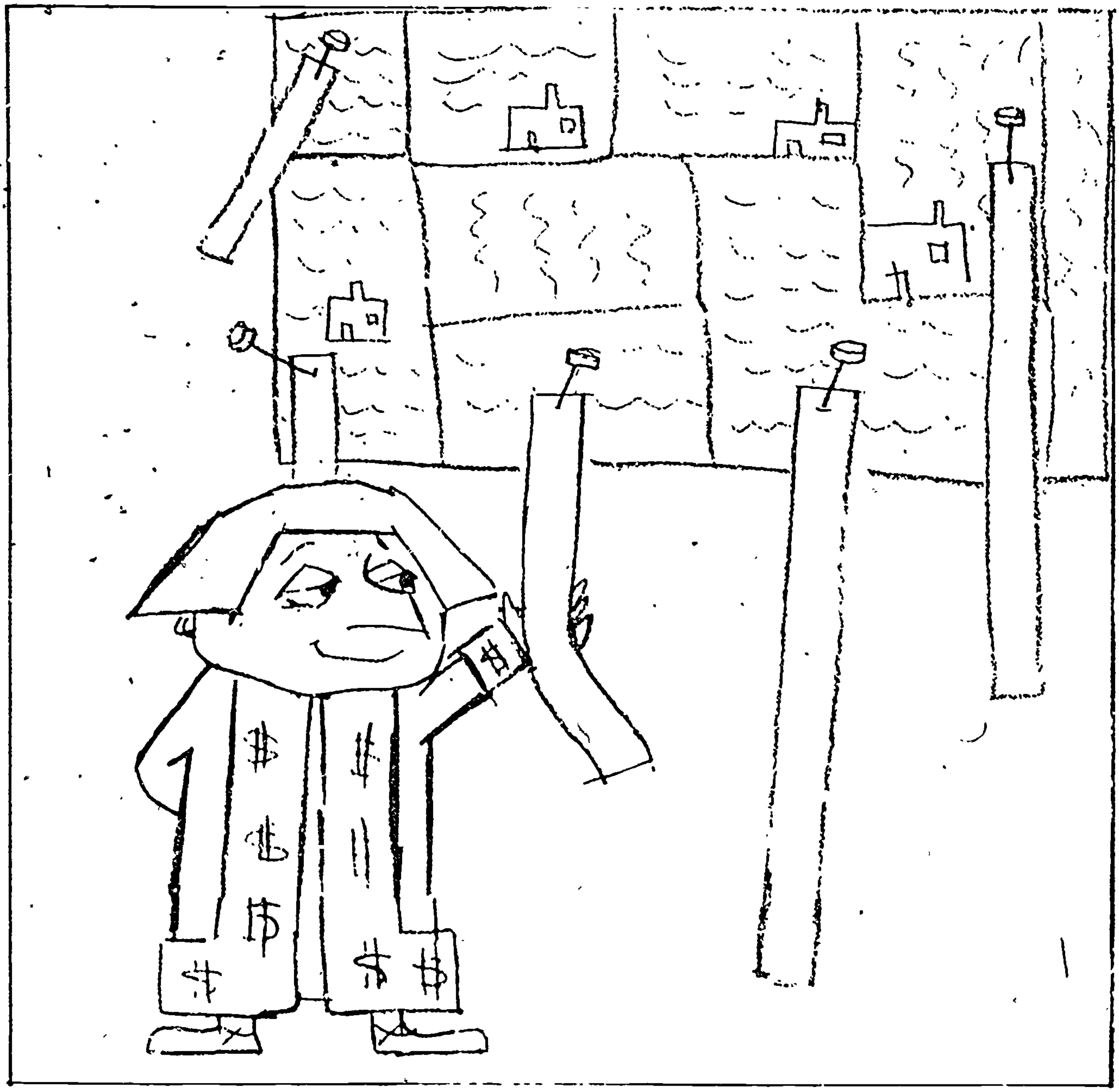


"But I was able to find a simpler method of comparing Egyptian farm fields. Here is a chart showing your field and the fields of your friends. You see how easily I can compare all three fields at the same time."

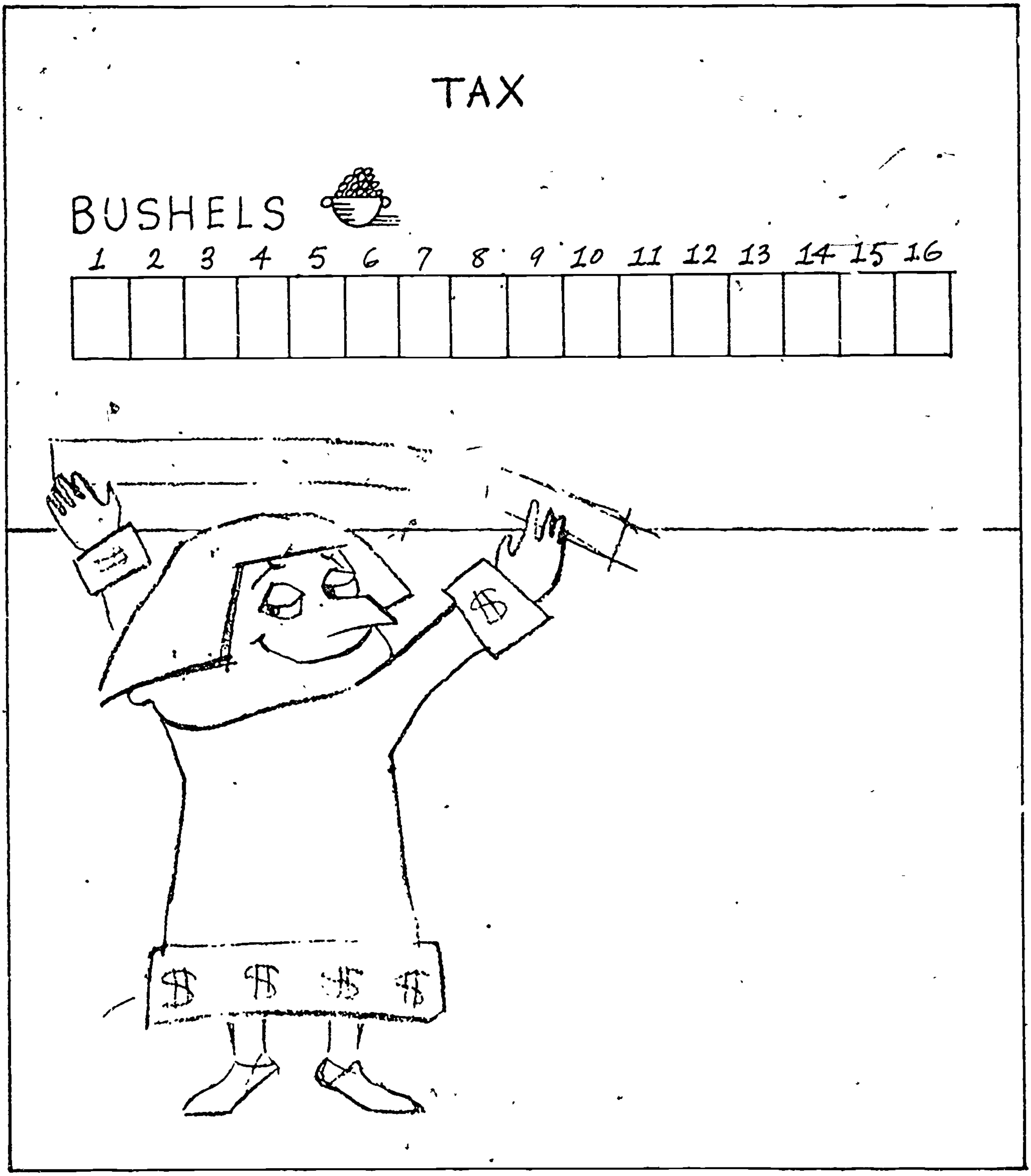




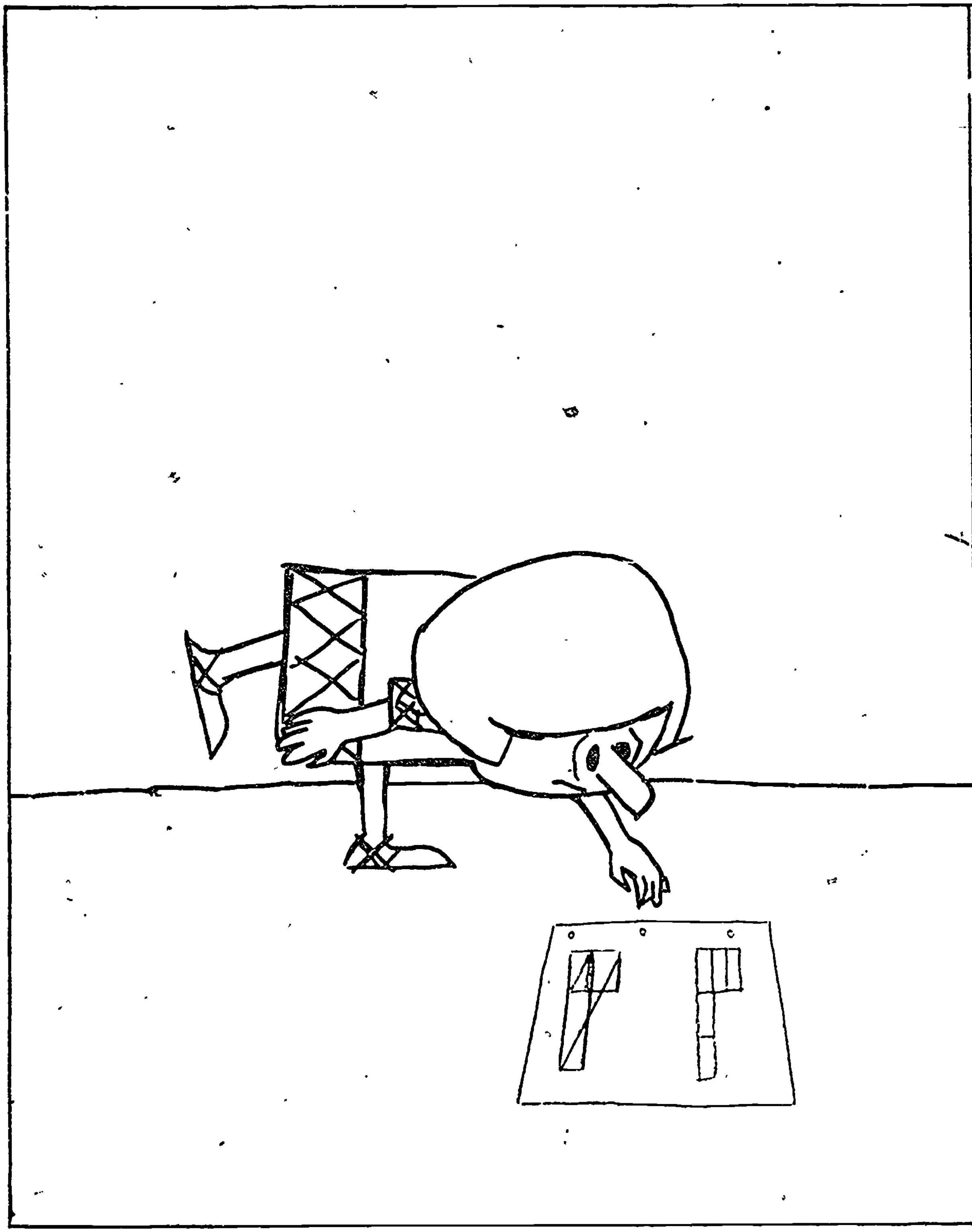
Ahmes was puzzled as to how the tax collector had made his chart.



In fact I keep records like this for all the land in Egypt. I have a strip for each farm. All the strips are the same width and the length of each strip depends on the size of the farm.



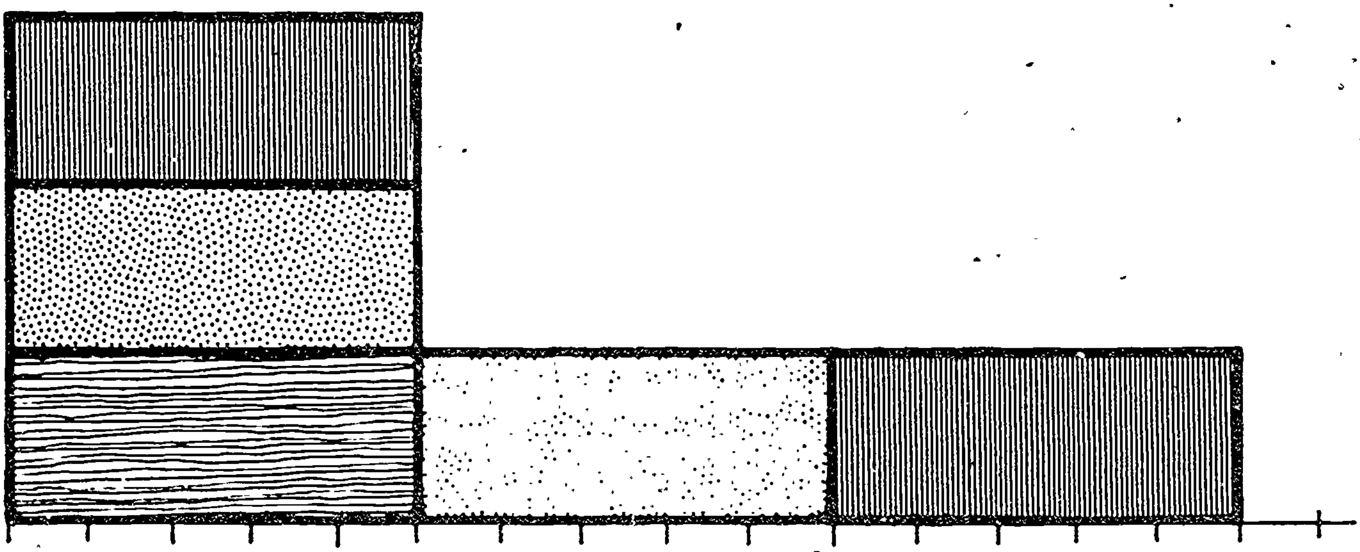
"To check how much tax each Egyptian must pay, I use this chart that tells me the exact tax, Ahmes."



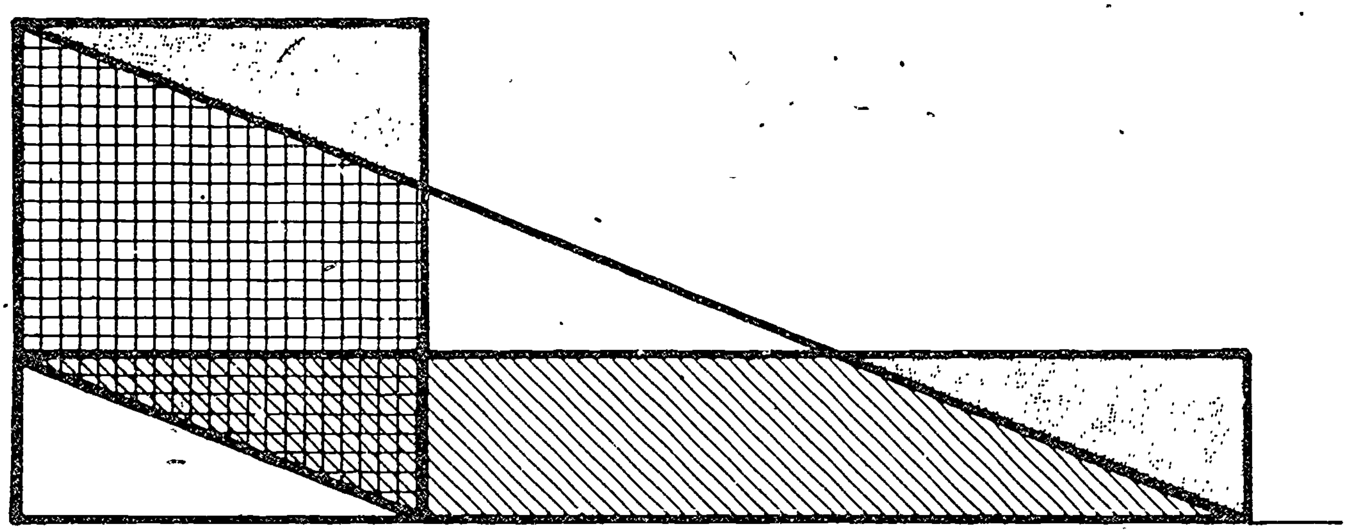
After the tax collector left, Ahmes found a page from his notebook lying on the floor. This page helped him to understand what the tax collector had done.

Two Ways to Get Strips from Rectangles

METHOD #1



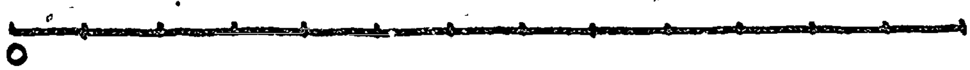
METHOD #2



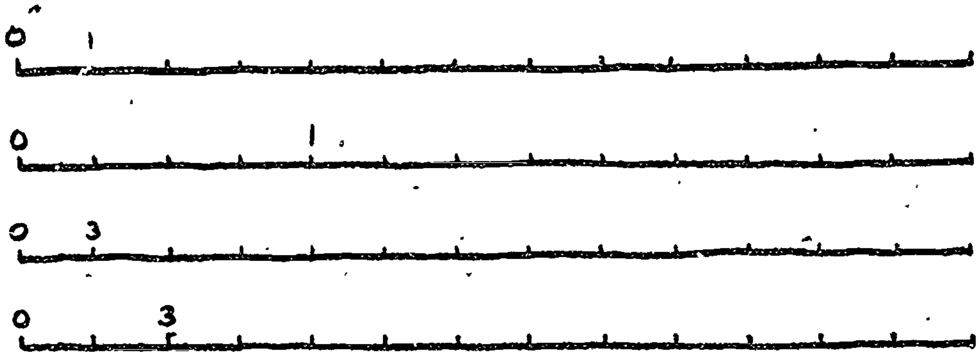
Here is the page from the tax collector's notebook which Ahmes found.

Paul also had a nice sensitivity for nomograms. He saw geometry as a clarifying agent for some of the difficult arithmetic concepts. His best application of this idea in my estimation came in his use of scales for multipliers. I will take you very quickly through a sketch of the steps he followed in a series of lessons in the intermediate grades.

1. First he introduced the equal division scale with zero. This same scale will be used throughout this discussion.

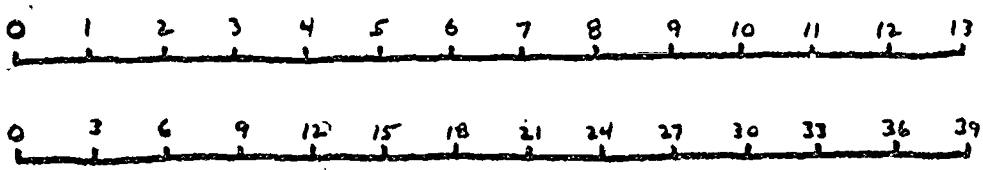


2. It was clarified immediately that placement of the next number determines the unit. We'll restrict ourselves here to non-negative integers. Some examples:



Filling in a scale once it is started in this way is straightforward.

3. Now scales are compared giving a physical model for multiplication

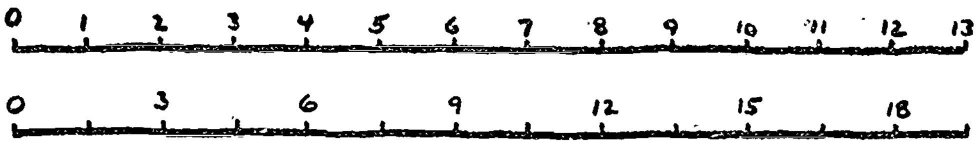


The zeros are "matched" and the pairing 1 $\longrightarrow$ a provides for any n
n $\longrightarrow$ na. Read down, the scales are an a-multiplier.

4. And of course division. On exactly the same scale pair we have $a \rightarrow 1$ and $n \rightarrow n \div a$. Read up, the scales are an a - divider.

5. Or multiplication by the reciprocal $a \rightarrow 1$ gives $n \rightarrow n \cdot \frac{1}{a}$.

6. With that introduction we have the full model for multiplication and division of rationals — or fractions as we're allowed once again to call them



Here we have

DOWN $1 \rightarrow \frac{3}{2}$ so down is a $\frac{3}{2}$ - multiplier.

UP $\frac{3}{2} \rightarrow 1$ so up is a $\frac{3}{2}$ - divider. But if we locate the one carefully on the lower scale we see that up is also $1 \rightarrow \frac{2}{3}$ so up is a $\frac{2}{3}$ - multiplier.

Since up plays two roles at the same time, $\div \frac{3}{2}$ and $\times \frac{2}{3}$, these are equivalent.

I did not always agree with Paul and I note some of those disagreements here. In doing so I carefully enter a disclaimer. It may well be that our discussions about these matters were askew, that we weren't communicating. I have not had time to forward a copy of this paper to him--something I will do shortly. What I suggest is that he may have a perfectly valid response or disclaimer or even change of heart.

The ridiculous first. In one of his primary grade books appears a circle, of diameter I recall about four inches. The child is asked to stretch a string around that circle and then....But of course at this point the lesson collapses. As I said — without success — to Paul, an adult cannot handle that task.

I also felt that Paul's attempt to bring the concept of real numbers to younger children led him into a trap. He wished to communicate this concept in the following way. Given two segments one of three things will be true of their lengths:

1. One will be a multiple of the other.
2. A multiple of one will equal a multiple of the other.
3. Neither 1 nor 2 will occur; that is, no multiples will be the same.

Of course type three introduces the incommensurables and the irrationals. No one here would question this two thousand year old concept, but I hope that some of you would join me in concern over attempts to have young children apply it. The problem is the unfortunate association of continuity with measurement. We are faced here with that everlasting dilemma: <u>In the real world there is no real number</u>. I see no way around this mensuration - continuity contradiction and our discussions went nowhere.

Here I will break my own continuity and foreshadow one of my comments about mathematicians in part two: I believe that the real number is the most difficult concept of mathematics through the second year of university. I suspect that any reasonable measure of understanding of this concept applied to students who have studied to that level would show less than a quarter even then with any handle on it. Yet Paul Rosenbloom and many others during New Math times sought to push this idea down into the grades. My alternate recommendation: postpone it until at least grade twelve and then hit it hard as a central idea of mathematics. Instead of communicating to our students that there are exactly three irrationals, π, e and $\sqrt{2}$, we might give them instead some feel for this major shift from the physical world to the conceptual world.

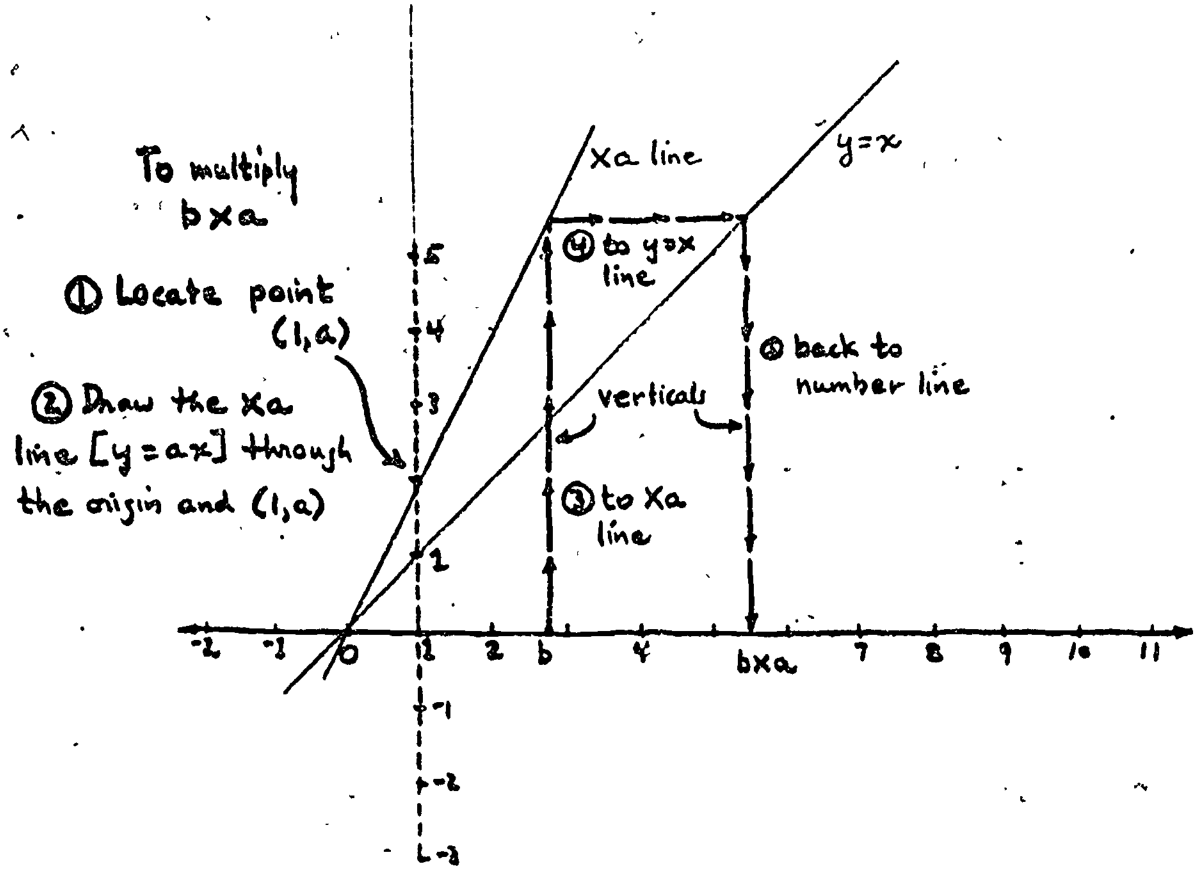
You may feel by now that I have spent too much time on this little remembered figure who both surfaced and sank as a contributing mathematics curriculum developer during the curricular upheaval of the late Fifties and early Sixties. I disagree with that reaction. To me Paul Rosenbloom represents by far the deepest thinking of the New Math times. He deserves this special attention. And he is also in many ways characteristic, perhaps even a caricature of the New Math mathematician - curriculum developer.

At any rate I will conclude my remarks about Paul Rosenbloom's work with an example that illustrates to me his profound thinking. I urge you to credit him with deep insight at the same time you prize out the right and the wrong of its application.

Paul criticized elementary math curriculum as it was in the Fifties — and of course is today in the late Seventies — for its collection of discrete and, even worse, often contradictory models. We have one model for multiplication of natural numbers: counting discrete objects. But that doesn't work when we extend to the integers so we create a new model. Then we need another for the rationals, still another for the reals. Confusing. What we need to replace this Babel is a single universal language. That language: the number line.

Now we all know the power of that model. I will be less than fair to Paul by not stressing some of the basic uses. Instead I will jump to the point at which he extended the idea dramatically, creatively — but too far.

The rule for adding reals (and by descent rationals, integers, etc.) on the number line is clear. We "add" segments. (I do not readdress the measurement problem here.) Now multiplication. Recall that a local model - 5×3 as 5 segments each 3 in length, added - won't do because it cannot be applied beyond the natural numbers. Here is Rosenbloom's model:



There are many important aspects of this model: It does indeed serve for multiplicatic ver the domain of the reals. You should explore how the algorithm responds to multiplication by a negative number and by 0. You should also demonstrate to yourself the product of two negatives. (Recall and make use of the fact that the $\times a$ and y = x lines extend through the origin.)

Surely you have already asked yourselves why the y = x line? Why not merely use the y-axis for the product? The answer is, it seems to me*, one of dimension. We want our answers on the same model from which our original numbers came. Consider this in the context of another popular model - popular to folks like Euclid, for example - area of a rectangle as

*I make no claim to represent Rosenbloom's own view adequately here.

a product. In that model we start with linear units and end with area units, something that poses more than a problem of cosmetics. There is also the problem of successive products. The Greek model will take us rather uncomfortably up to three dimensions; from there we must temporize. Paul's model works for multiple products directly if not easily. You may wish to test his technique, for example, for proofs of associative and distributive laws.

A beautiful conceptual model. I go further: Paul developed the beginning of an attractive pedagogical approach to this model in the middle grade MINNEMAST units related to Squaresville. Squaresville is a city with numbered streets and avenues like New York and Minneapolis. New streets are constructed cutting Broadway-like cross town, Equality Boulevard for example. Some might criticize this model in the name of purism using Paul's own terms: as applying only to natural numbers. I would not. Here the restriction is associated with pedagogy.

So what's wrong? Two things, I think. First the model is difficult, even impractical except over a very restricted range of numbers near zero. Try multiplying by six, for example. This is not necessarily a reason for rejecting the model, by the way; it does, however, severely restrict its pedagogical value.

The second problem is one that is, I suppose, more or less true of any model. The problem has been nicely summed up in the appelation "parachute postulate". This expression I first saw used for what is surely the definitive case, by the Cambridge Conference in reference to the use in the SMSG eleventh year program of what amounts to the integral definition for the logarithm function

$$\ln x = \int_1^x t^{-1}\, dt$$

The applicable question is, of course, where the hell did that come from?

SMSG has always been representative in the minds of most Americans of New Math hot from the mathematician's cow. But SMSG is most difficult to pin down. It is very much like a dog I had when my children were young: a mixed breed. (Some would say that the simile would carry further: the part of my dog that was chow was the mouth. She bit thirteen people.) The writing teams for different grade levels were essentially discrete. In only the case of the first of the several geometries was a single author a major force. The source of that force was, of course, the powerful personality of Ed Moise.

The mathematics of that geometry is, I believe, impeccable. The course itself is indeed a tour de force. I had the great pleasure of having a Moise student teach this program in Norwalk when I was supervisor there. His sensitive handling of this text made clear to me the sense of the Moise comment I quoted earlier.

But in the hands of others it often was - and still is today - a mess. Before turning to what are some of the reasons for that I note the interesting parallel between the Moise-Birkhoff approach and the Shanks-Hilbert approach of the Ball State geometry. Others have commented on content differences between these two programs: what might be worth exploring as an alternative are the personality differences between these so unlike mathematicians. Leading, I must note, to the same unsatisfactory end result.

Much praise has been directed to the SMSG teacher's manuals which for the first time provided content equal in volume at least to that of the texts they were designed to support. I join in that praise. For teachers who used them - too small a proportion: time is ever a factor in the teacher's lesson preparation - those TMs played the role of a well-focused in-service mathematics course. All to the good: good teachers could use this support as they undertook programs new to them. What these manuals failed completely to provide, of course, was pedagogy.

An example from the SMSG ninth grade algebra: The student text presents the rules for addition of integers formally by affixing appropriate signs on natural numbers. For example:

$$\text{For } a > 0,\ b < 0,\ |a| < |b|,\ a + b = {}^{-}(|b| - |a|)$$

Now when I went to school the rule that covered that case and one other was given as "when the signs are unlike find the difference (in numerical* value) and affix the sign of the larger (numerical value)." Interestingly I find the SMSG rule pedagogically worse than the old rule because it superimposes notation on nonsense. Here I use nonsense in its strict and non-pejorative meaning of something that is not reasonable. What is missing is how to teach this incomprehensible Greek.

Let me assure you how it was usually taught: as rules to be memorized without explanation. End of career for many young mathematics students.

While you mull over that claim I invite you to compare that insensitivity on the part of teachers with some of the parallel insensitivity on the part of some of your own university teachers. Did you ever have one who seemed to feel that making sense of (university teachers call it motivating) what he was delivering in the most formal of terms was not his concern. In this regard I recall the university algebra class I audited several years ago at my own institution in which the teacher was teaching — I use the word loosely — the idea of isomorphism. I am convinced that no student in the room came away with any sense of what this important concept was about. The teacher never even tied it to the one example that was within their experience: logarithms.

*Note the use in many old books of "numerical" for what today we designate "absolute".

I believe that SMSG writers operated from an invalid assumption and that that is what caused their imposing edifice to crash down on so many of our youngsters. Their assumption: Classroom teachers knew pedagogy; they just lacked content. They assumed that the classroom teacher would translate their good math into viable classroom lessons.

The effect of this on secondary school instruction was bad enough, in the elementary schools it was tragic. Elementary school teachers, worried about their competence to teach the new ideas would go to school themselves to master that new content. In a few cases they were well-taught — often at the level they would then teach their students. Too often, however, they were taught CUPM math: formal, university level approaches to the content. I have seen the result in its most virulent form. Join me as I sit in a primary classroom watching a teacher tie up a beautiful reading lesson. She now turns to math and lectures. Insecure, without assistance or guidance, she is duplicating at the rote level how as well as what she was taught. Wrong? Of course. Her fault? I claim not. The mathematician's fault: yes (for poor and insensitive teaching) but not entirely, for where were we? I'll tell you where we were: we were hovering around the fringes trying to be little mathematicians just as today we — and everyone else in education — try to be little psychologists. We have failed as abominably in both roles.

Not everyone. Enter Max Beberman, Bob Wirtz, et al., people who took serious ideas from old math and new and ran with them pedagogically. Beberman's work has been often cited, not nearly often enough perhaps and too often the wrong things like the number-numeral fetish from which he himself turned away. His creative approach to word problems is well worth notice but is essentially forgotten today. But I will attempt to characterize these translators with just one example from Bob Wirtz, quite frankly one of my heroes.

SUMAS DE SERIES SUMS OF SEQUENCES

14=2+3+4+5 15=7+8 45=14+15+16 39=19+20

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, ...

15=4+5+6 50=8+9+10+11+12

1=	19= 9+10	37=
2=	20=	38=
3=1+2	21=	39=
4=	22=	40=
5=	23=	41=
6=	24=	42=
7=	25=	43=
8=	26=	44=
9=	27=	45=
10=	28=	46=
11=	29=	47=
12=	30=	48=
13=	31=	49=
14=	32=	50=
15=	33=	51=
16=	34=	52=
17=	35=	53=
18=	36=	54=

How does Wirtz generate such ideas? I believe that his method is this: He organizes exactly what he wishes to teach, not limiting himself to product goals, then he reviews kinds of things he has seen others do related to the task at hand, and finally he draws on his own creativity to put those things together or even to develop entirely new activities designed to teach what he set out to accomplish. In the example I have just used, I think Bob set out to provide some experiences with addition in a richer environment than mere computation. Most of us, I think, miss the point of this kind of lesson. We focus on the problem solving aspects of what is presented here and are led quickly up Bloom's cognitive taxonomy to such activities as synthesis and proof. All well and good, but that is all in the nature of a bonus: this is still an activity whose primary focus is on computation. Bob's title is wholly appropriate here: Drill and Practice in a Problem Solving Setting.

Bob is, I believe, the absolute best at this. His curricular materials are loaded with similar examples of the activities that we too often forget or even avoid - and that we are quite correctly under fire for by the Back to Basics crowd - such activities raised by the quality of his presentation to a level at which they are attractive to us all.

I have already named others who have been good at this. Let me associate some of them with a few of their specific products. Peter Braunfeld: the UICSM Stretchers and Shrinkers and the CSMP Book 0 development of decimals. In these materials Peter has taken prosaic tasks and like Wirtz addressed them in creative ways. The Braunfeld approach is different in that it extends the same activity over a series of lessons. Surely we need not argue the merits of one of these approaches over the other; each has advantages. Zed Dienes has provided concrete activities that address very basic logical goals. Like Piaget he has seen through and beyond our focus on content superficialities. It is difficult to select from Arthur Engel's rich contributions, but I believe that his development of probability in the CSMP Book 0 is his best effort.

Harold Jacobs is worth special note. Unlike the others I have mentioned he has been able to write directly for commercial publication. His Mathematics: A Human Endeavor and Geometry demonstrate the fact that you can get good content into print. Interestingly these books are too rich for many teachers. This is a case of the text outdoing the teacher and essentially leaving him no role. I don't buy that as anything but a criticism of those teachers for two reasons: (1) If that were the case, they could give their students a more standard text (or none at all) and use Jacobs as a lesson source. They don't. (2) Jacobs himself addresses this problem. The teacher's manual for Mathematics: A Human Endeavor is the second best book of its type. It provides plenty of parallel and additional rich activities for the teacher to use. I am afraid that this is again a case of our inability often to go beyond leading the horse to water.

*

Those then are some of the all too few translators of quality mathematics into rich curriculum. It is as I write this a great temptation to idle here for hours displaying the rich wares of these creative people. But talking about them further would divert us from my basic task: to explore the mathematician's contribution to curriculum development. What I have said needed to be said in order to distinguish the role of curriculum developer from that of developer of the mathematics itself. To go further would be to make too fine a point of a demarkation that tends to fade under closer scrutiny. The one thing to take from all that has gone before is the fact that only Rosenbloom of all the New Math workers in the Western Hemisphere completely straddled the two fields.

Now, however, we come to the absolute top of this precious heap. It makes me uncomfortable to keep raising the pitch of my praise, especially given my long-standing record of harsh criticism, but I now introduce the best of the best - la creme de la creme.

*

George Papy and Frederique, I will consider as one. I believe that they themselves would prefer that. In this unique combination we have the best of both worlds: a creative mathematician and a highly creative teacher. Beware. Do not make the mistake of assigning to George the first role and Frederique the second. In them the two roles both overlap and interact so that such a separation is not only non-contributory but also inaccurate.

I have rather artificially separated the Papys from New Math by my restriction to the Western Hemisphere of my earlier remarks. What some fail to recognize is the fact that the Papys together developed both a secondary school program (first) and an elementary school program in Belgium before they came to work at CSMP in the United States. Unfortunately only a few of these books - four I believe, two at each level - have been translated into available English texts.

I will comment very briefly on the Belgian secondary school program and then turn to CSMP. The two volumes of Modern Mathematics published by Macmillan in English at first examination appear as insensitive to reality as the Cambridge Conference goals. It takes not only careful and detailed examination but also direct classroom trial to convince a teacher that this content is not only rich but pedagogically viable. Here is New Math the way it should have been translated for students.

Now Frederique is completing the CSMP K-6 elementary school program: * at the same time the richest mathematics program for this level that exists today and the richest pedagogical program for this level that exists today. One aside about the program is worth while: It is used by over half the primary school population in Philadelphia. I mention that only to dispell the widely circulated false characterization of this program as - like the secondary school CSMP program - only for gifted students.

I will spend some time on the CSMP elementary school program for which Frederique is the major creative force and Papy a senior consultant. I give this attention for several reasons, not the least of which is that

* Clearly the CSMP program is the result of work by its full staff; I have chosen here, however, to assign the creative role to the Papys. This is accurate in general but not in detail.

CSMP is fun. Here is what New Math should have been. And it is too little known: few even of you know it well; some, I am sure, have never heard of it. It is also conceptually organized and has a pedagogical point of view.

This last is important and is worth exploring further. The CSMP staff has always held the mathematician in a position not only of authority but even of awe, not just in his designated sphere of activity but in curriculum development and pedagogy as well. I have fought this elitist posture over the years, offering such examples as Ed Begle and John Harvey to make my point. The response is essentially that those are exceptions that prove the rule.

But I will have to admit that in both the secondary and elementary CSMP programs the mathematicians offer a strong argument for their case. In particular the elementary program addresses the curriculum quite differently. "Here is what can be done when a mathematician really puts his mind to it," is the clear message of this program. I must add, of course, that the difference is not just from what non-mathematicians have done but from what mathematicians have done elsewhere as well.

What is so important here is the mix of mathematics and pedagogy which are so well homogenized that it is not possible - for me at least - to separate the two. This metaphor is worth applying to so much that passed for New Math, where the heavy cream of content rose to the top out of the skim milk of pedagogy and effectively clogged the neck of the delivery bottle. (Among other weaknesses of that metaphor is the fact that it clearly dates me.)

I will display some examples of CSMP content in a moment, but first I briefly outline some major ways in which this program differs from standard school content at any level. There are two of these differences, one clearly stated in CSMP's own descriptions of their program, the other not. The unstated difference is so important that I take it first.

What is the usual approach to math? I believe that this approach is so much a part of our tradition that, given the opportunity to tell how we would organize a teaching program from scratch, each of us would say essentially the same thing. First we decide what content we wish to teach Then we break that content down into linearly ordered pieces for presentation: ordered in the sense of mathematical dependence. In most developments, for example, counting precedes addition, group axioms precede field axioms, exponents precede logarithms. Curriculum development in the past has, I submit, been largely a matter of tinkering with this approach: changing the order, perhaps (logs before exponents), or inserting a few new topics.

So we have in essence the freight train curriculum, a long series of cars, each a day's work, with the curriculum worker the switchman at the railroad yard where the long trains are built up.

The Papys have rejected this model and provided one that is much more in tune with the teaching, say, of literature. They address very general goals, the day-to-day goals of the standard approach achieved as necessary and appropriate fall-out. Thus today's lesson might be on some calculator problems but seldom on the next step in an algorithm. I say seldom because perhaps ten per cent of the curriculum does take this form, but even there an important difference is evident. That ten per cent comes at the end of the more general explorations and other activities rather than at the beginning.

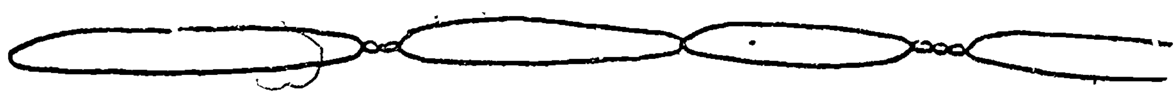
I apologize here for stealing over briefly into my colleague Professor Weinzweig's area of concern (especially since I think that his topic is essentially vacuous - his topic, that is, not his talk) to comment briefly on this. The freight train model seeks pedagogical closure at each step of the way. As you clamber along through or over the cars - or as more often you watch them pass you by - you are supposed not only to absorb the content but also to build on it. One aspect of this model that is particularly bad is its failure to provide room for discovery, for a growing power to attack original ideas, for participation in learning.

What we had in New Math, of course, were Davis's and Page's nice discovery activities essentially independent of the curriculum, but with the Wirtz exception none addressing the basic content of school math.

If you give students an algorithm, they will sit still for applying it but not for such things as exploring how and why it works, developing alternates, or in fact much of anything involving independent thinking. This is exactly what is at the source of the very narrow essentially trivializing view of math held by the general populace and indeed some mathematicians. In CSMP, on the other hand, with algorithms delayed - not by minutes or hours but by months - students are forced to seek out and develop their own ways - read if you will algorithms - to attack problems. Then when pedagogical closure is finally reached in that delayed ten percent those earlier explorations are not discharged but instead form a basic process attitude toward math. This is, I claim, a very important pedagogical difference from the norm, worth exploring in great detail.

So the first way CSMP differs is in its replacement of the freight train curriculum model

with a more global model of curriculum:



The second major difference between CSMP and more standard programs is found in CSMP use of what they call pedagogical languages, thus curricular vehicles for communicating the mathematical ideas. The CSMP pedagogical languages include:

- the language of strings - Venn diagrams
- the language of arrows - unary operation and function diagrams
- the language of the minicomputer - a decimal-binary calculation tool that gives calculating and problem solving power early

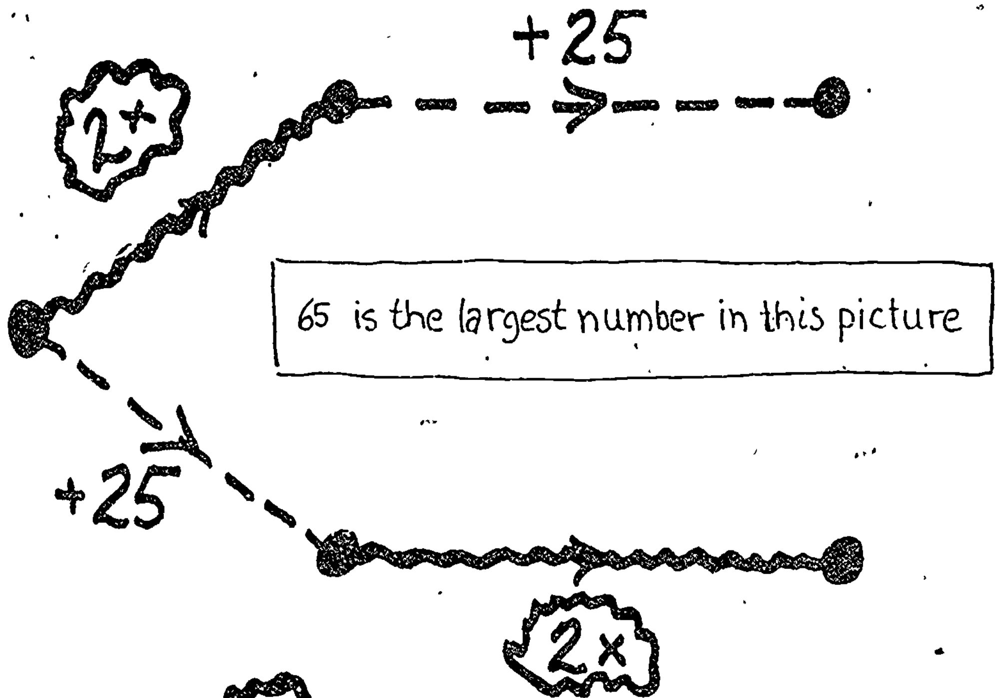
- (the language of) the hand-held calculator
- (the language of) the detective story
- (the language of) the translator - a parallel ruler that contributes to notions of parallelism, symmetry and vectors

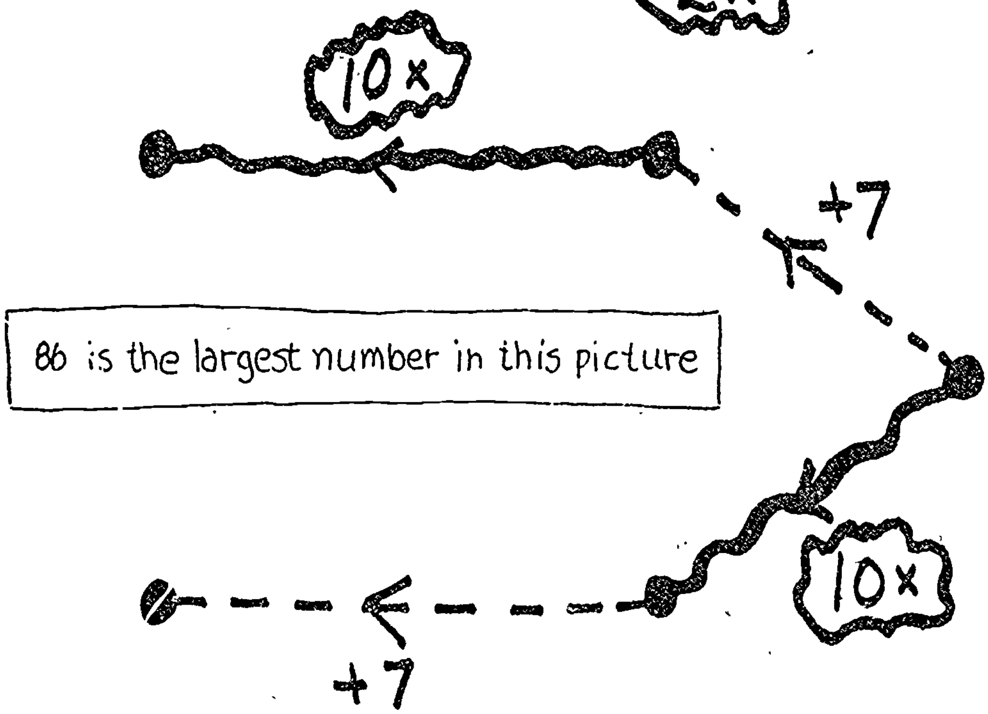
I will try to communicate to you some of the power of several of these languages in the following examples.

Name ______________________ N 03 ***

All the dots are for positive numbers.

Label all the dots.

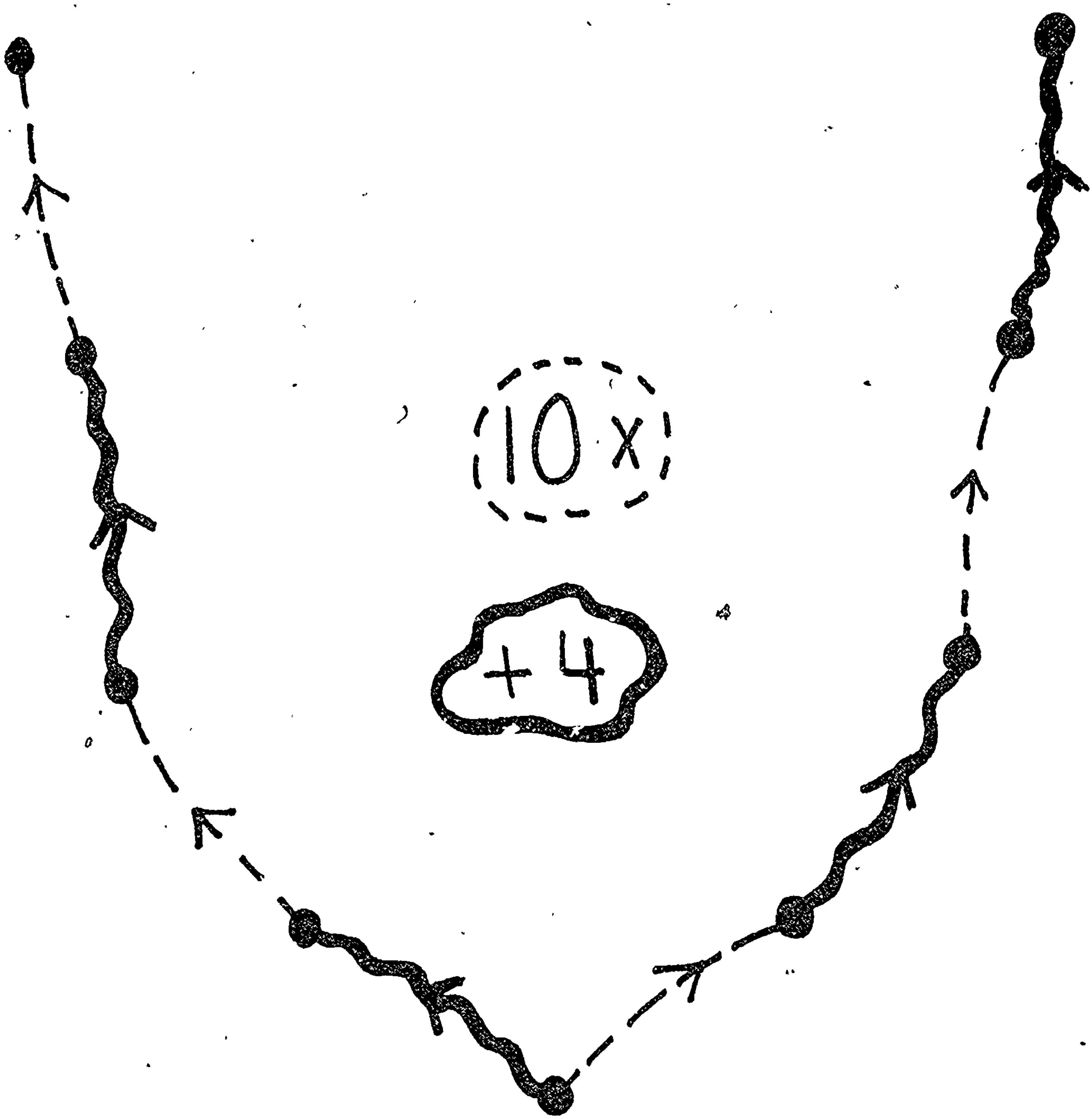




Name ______________________ N 07 ***

In this picture, all the dots are for **POSITIVE** numbers and 685 is the largest number.

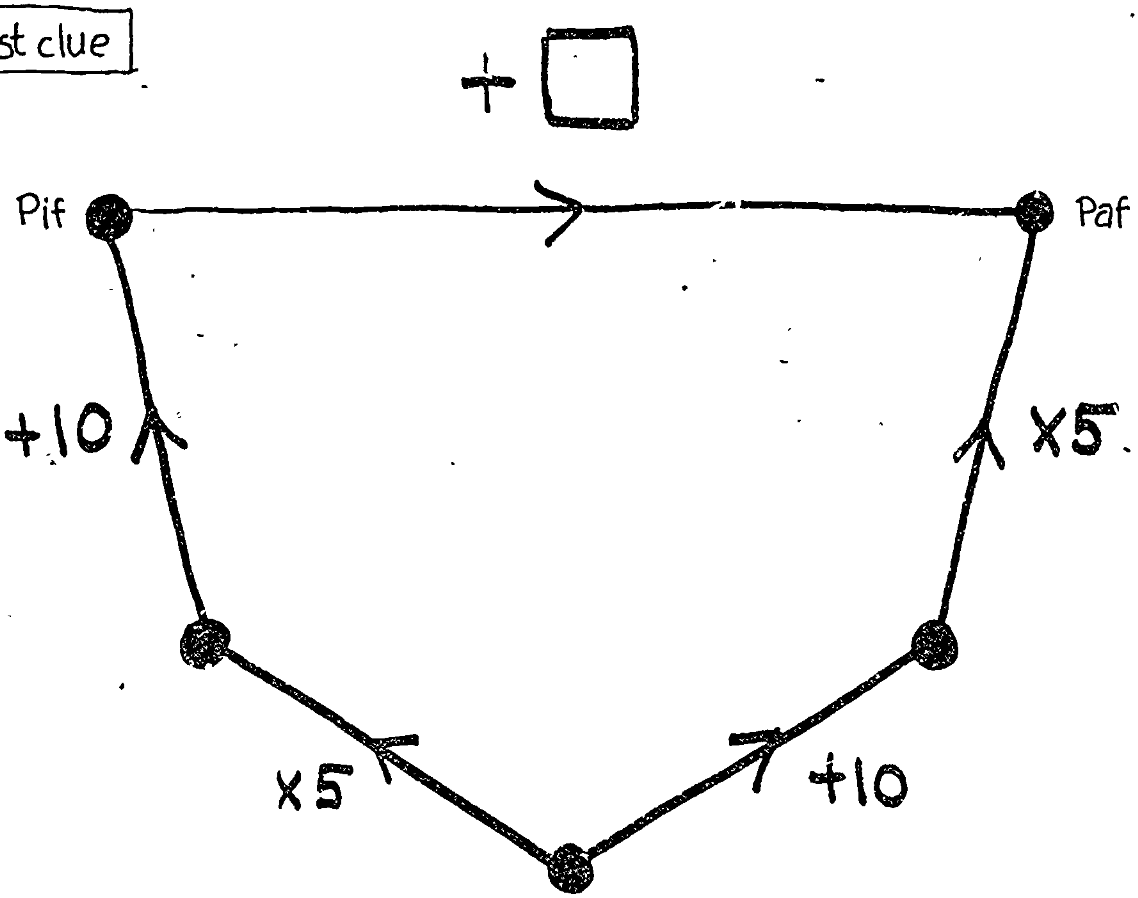
Put 685 in the picture and label the dots.



Name ______________________ N 08 ****

Pif and Paf are two secret numbers.

First clue

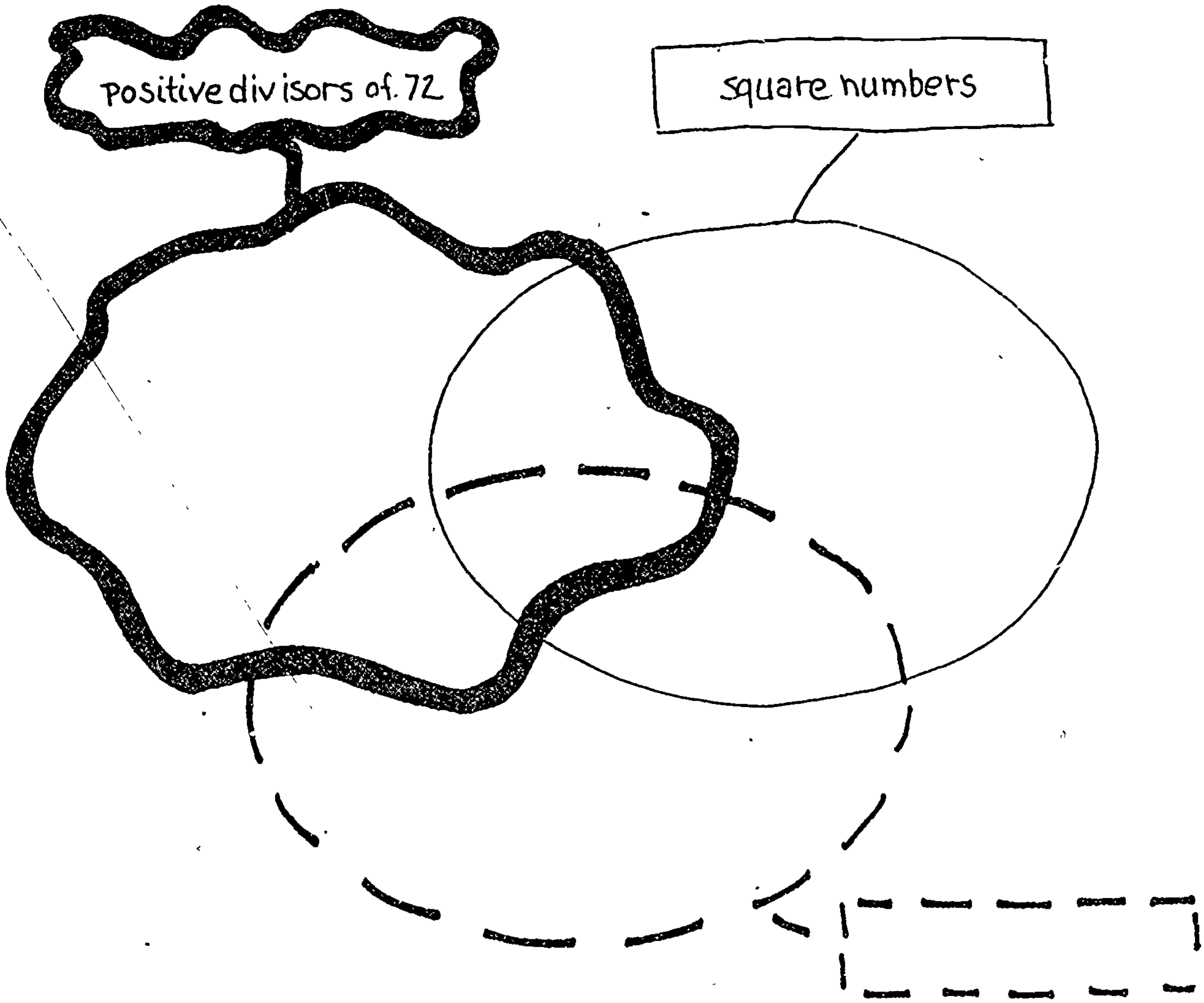


Second Clue

Paf is the double of Pif

Pif is ________. Paf is ________.

Name ______________________ N 12 ****



the empty tag is one of these:

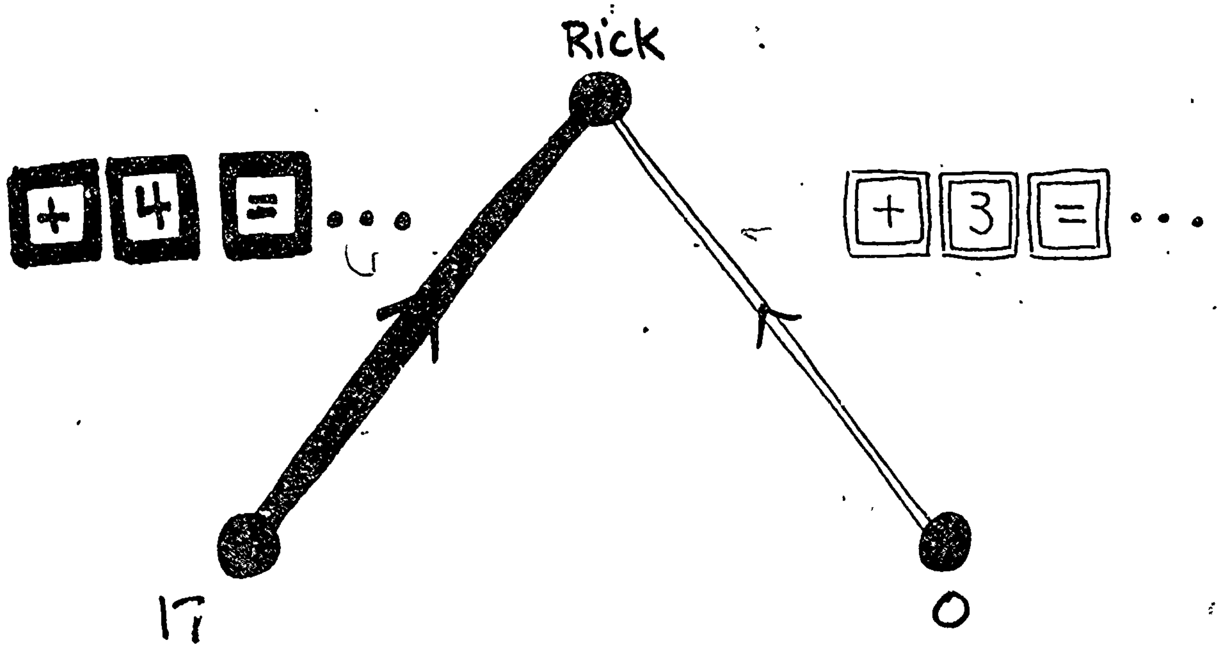
With this information you should be able to place seven of these numbers in the string picture

2; 4; 9; 15; 20; 49; 60; 64; 72.

Place them.

Name ________________________________ N16 ****

First clue



Rick could be ___; ___; ___; ___; ___; ___; ___; ___; ___;

___; ___; ___; ___; ___; and so on.

Second clue

Rick is between 1,200 and 1,220.

Rick is ____.

Name ______________________ N 17 *

Jo is a secret number

First clue

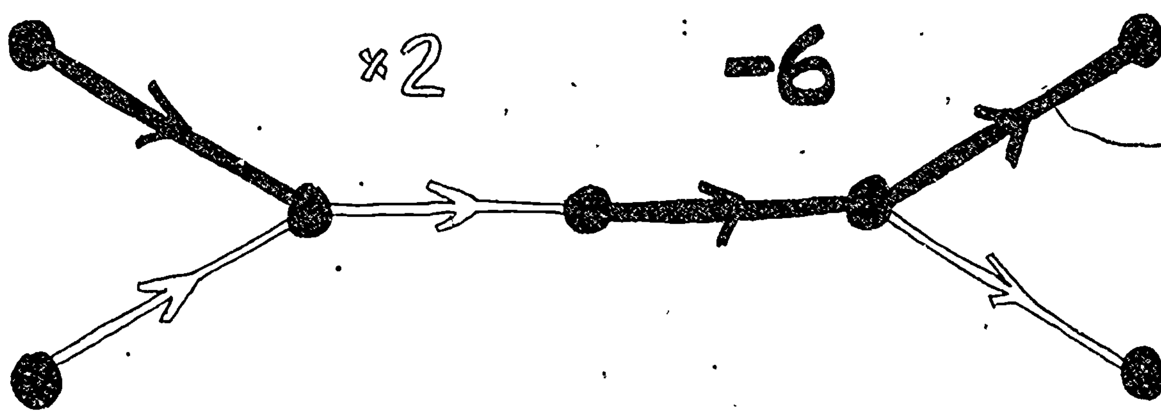
One of the symbols: +, −, × is hidden in each blank of this hand-calculator sentence. A symbol may be used twice.

7 [ ] 3 [ ] 2 = Jo

Jo is ____ or ____ or ____ or ____ or ____ or ____ or ____ or ____.

Second clue

Jo is in this arrow picture



Jo is ______ or ______.

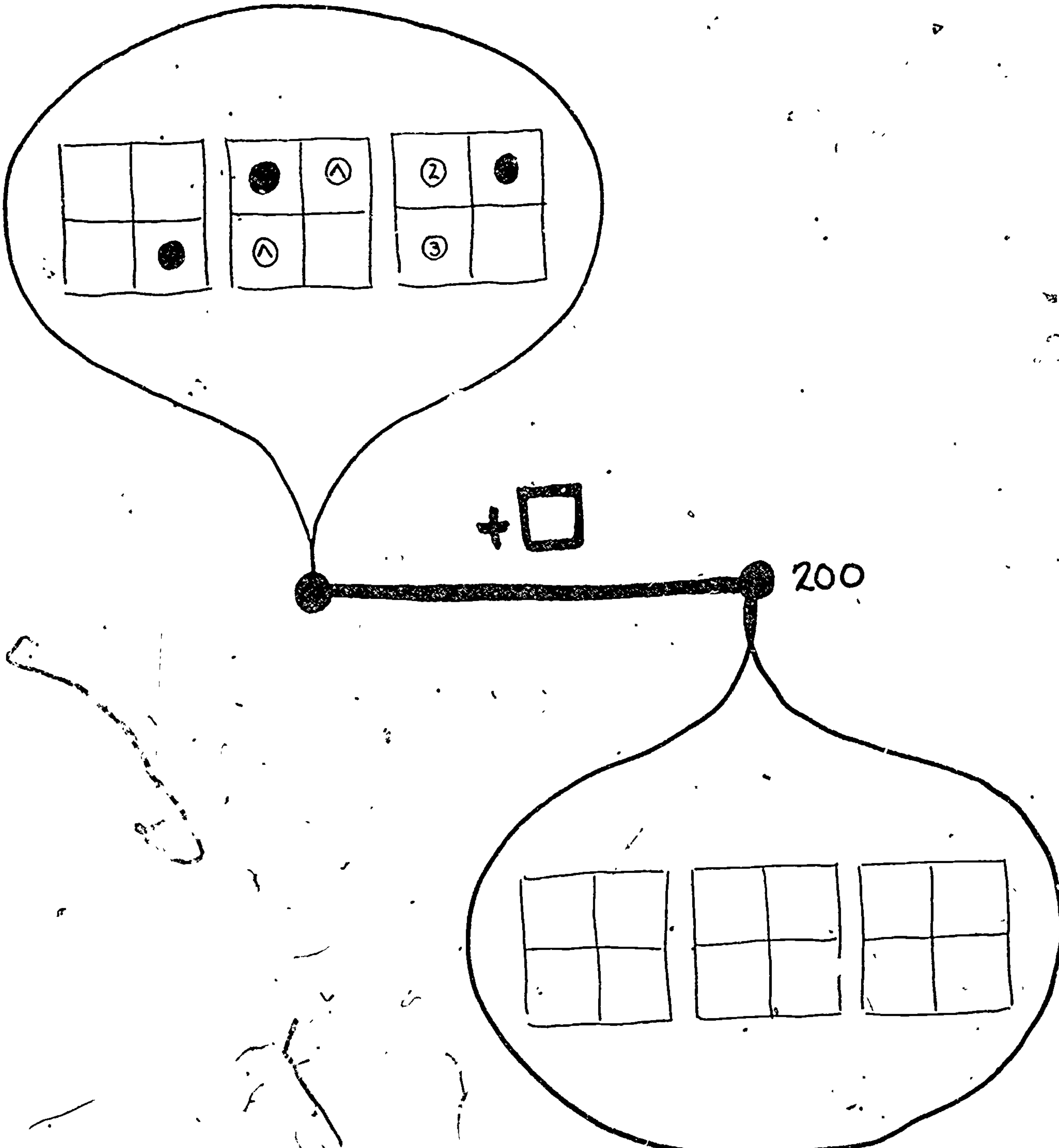
Third clue

If you add 1 to Jo, you will get a square number.

Jo is ______.

Name ______________________ N 33 ***

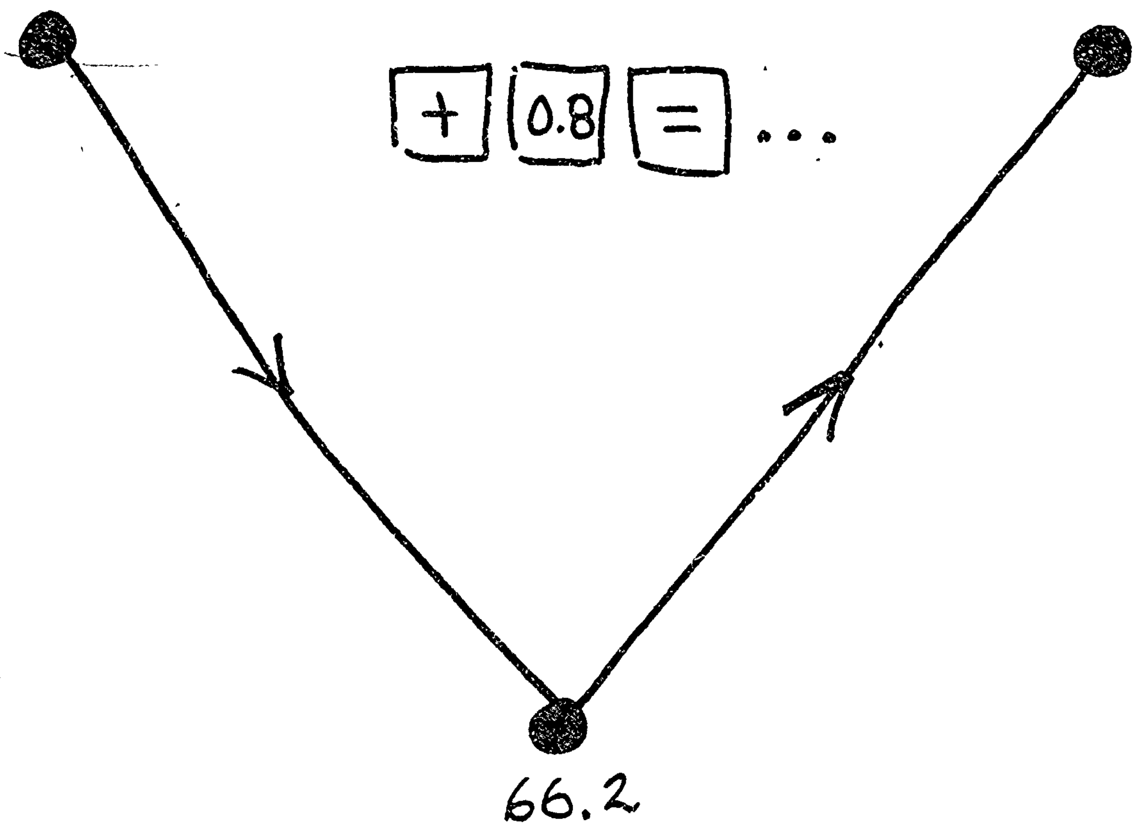
Solve this puzzle by moving exactly one checker.



Name ______________ N 38 ****

Kri is the smallest positive number which could be here.

Kra is the smallest number larger than 1,000 which could be here.



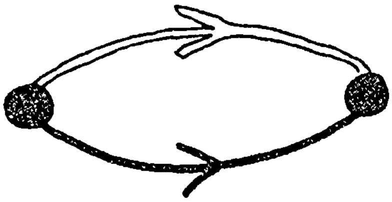
Kri is ____.

Kra is ____.

Name ______________________

Label the dots.

$\frac{3}{2}\times$

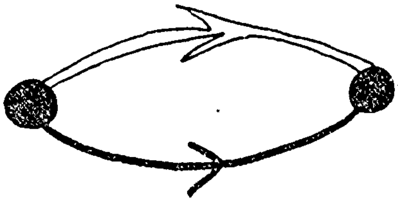


+50

$\frac{2}{3}\times$

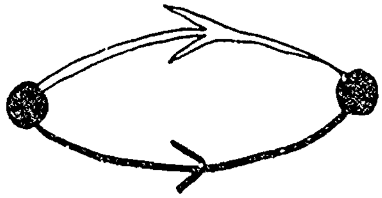
−40

$\frac{4}{5}\times$



−20

$\frac{3}{4}\times$



−30

$\frac{6}{5}\times$

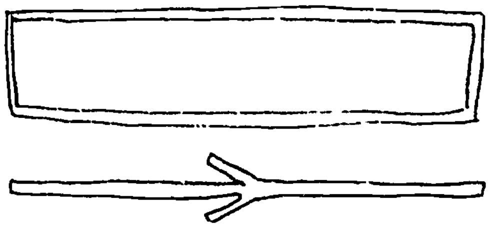
+15

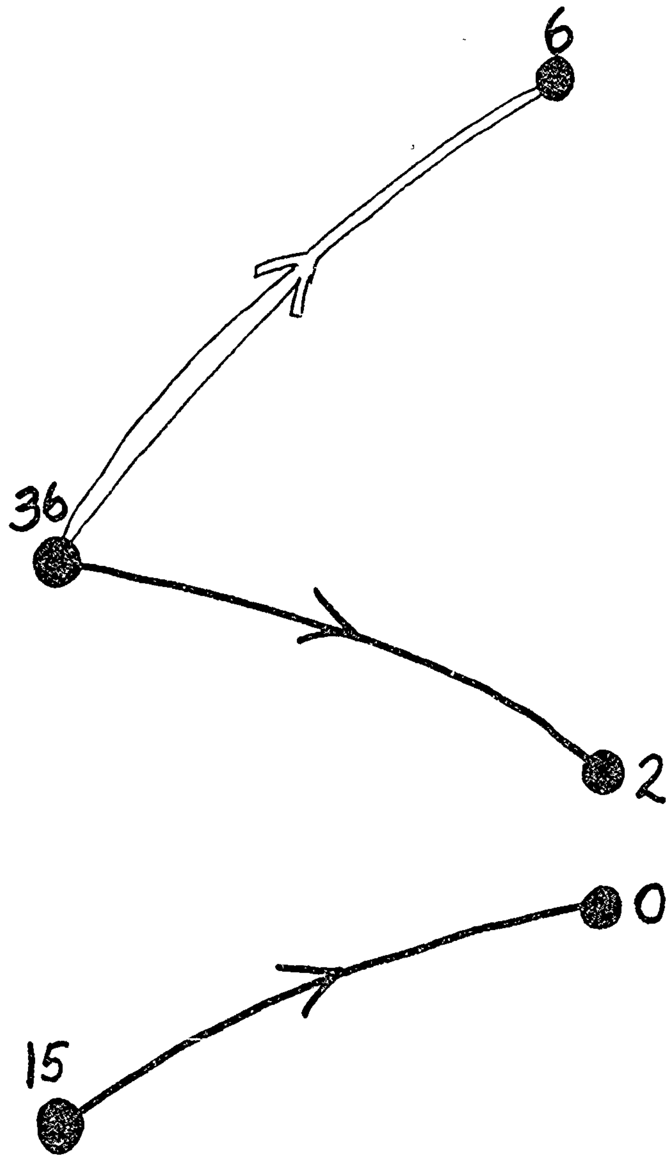
$\frac{5}{6}\times$

−25

Name ______________________________ N 90 ***

Complete the arrow label.

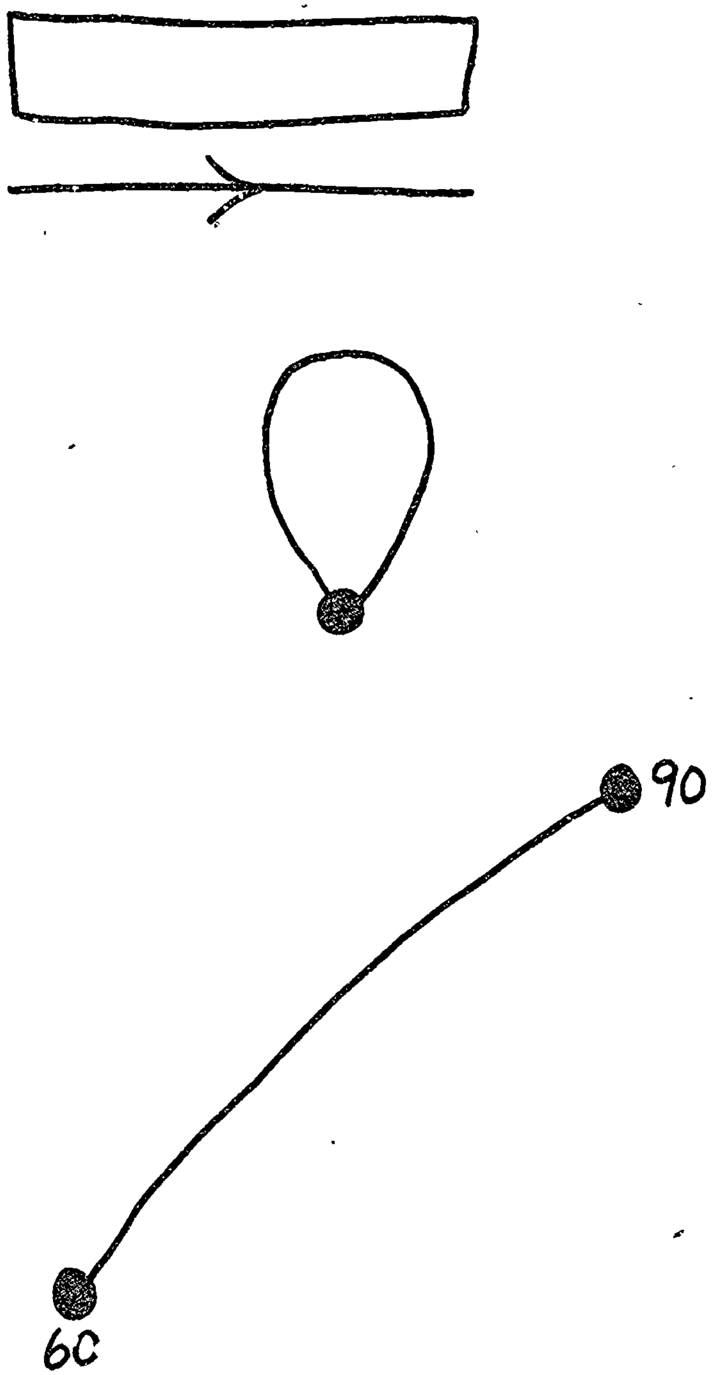




The outlined arrow could be for:

is larger than
is smaller than
is at least 20 smaller than
is at least 20 larger than
is a multiple of
is a positive divisor of
is the square of
$10\times$
$\div 10$
$2\times$
$\div 2$
$\frac{2}{3}\times$
$\frac{3}{2}\times$
$\boxed{+}$ $\boxed{=}$ ...
$\boxed{+}$ $\boxed{2}$ $\boxed{=}$ ...
$\boxed{-}$ $\boxed{2}$ $\boxed{=}$ ...
$\boxed{+}$ $\boxed{3}$ $\boxed{=}$ ...
$\boxed{-}$ $\boxed{3}$ $\boxed{=}$ ...
$\boxed{+}$ $\boxed{4}$ $\boxed{=}$ ...
$\boxed{-}$ $\boxed{4}$ $\boxed{=}$ ...
$\boxed{+}$ $\boxed{5}$ $\boxed{=}$ ...
$\boxed{-}$ $\boxed{5}$ $\boxed{=}$ ...

Name ______________________________ N 91 ****

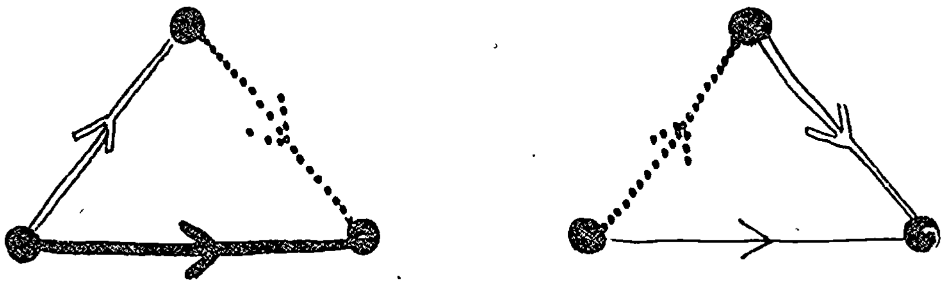


the black arrow could be:

is larger than
is smaller than
is at least 20 smaller than
is at least 20 larger than
is a multiple of
is a positive divisor of
is a square of
10 x
÷ 10
2 x
÷ 2
$\frac{2}{3}$ x
$\frac{3}{2}$ x
$\boxed{+}$ $\boxed{=}$ ...
$\boxed{+}$ $\boxed{2}$ $\boxed{=}$ ...
$\boxed{-}$ $\boxed{2}$ $\boxed{=}$ ...
$\boxed{+}$ $\boxed{3}$ $\boxed{=}$ ...
$\boxed{-}$ $\boxed{3}$ $\boxed{=}$ ...
$\boxed{+}$ $\boxed{4}$ $\boxed{=}$ ...
$\boxed{-}$ $\boxed{4}$ $\boxed{=}$ ...
$\boxed{+}$ $\boxed{5}$ $\boxed{=}$ ...
$\boxed{-}$ $\boxed{5}$ $\boxed{=}$ ...

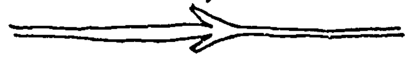
Name ______________________ L03 ***

Complete the table.

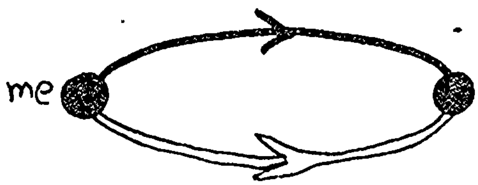


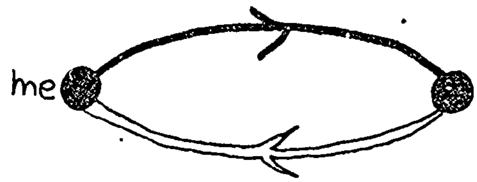
R	S	RoS	SoR
−6	−13		
5x	4x		
3x		6x	
	−7		−4
8x		4x	
you are my son	you are my sister		
		you are my maternal grandfather	
you are my daughter	you are my husband		
		you are my friend's brother	
you are 5 years older than I	you are 3 years younger than I		
you are older than I.	you are the same age as I		

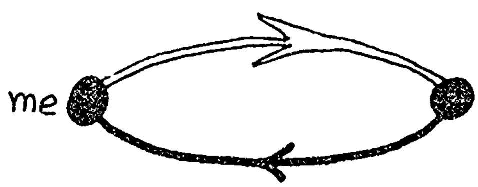
Name ______________________ L 08 *

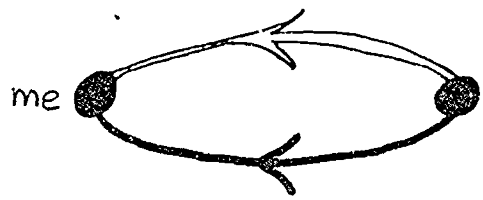
You are my mother.	You are my teacher
	

Decode:







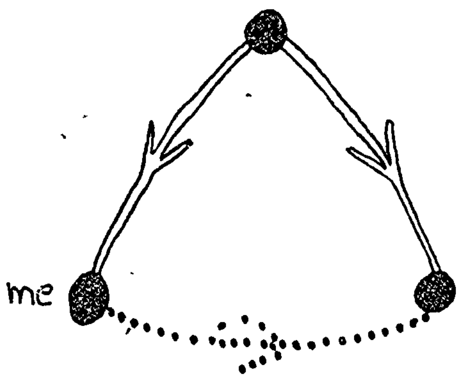


Name ______________________ L 10 ***

You are my mother

You are my teacher

What could the dotted arrow be for?



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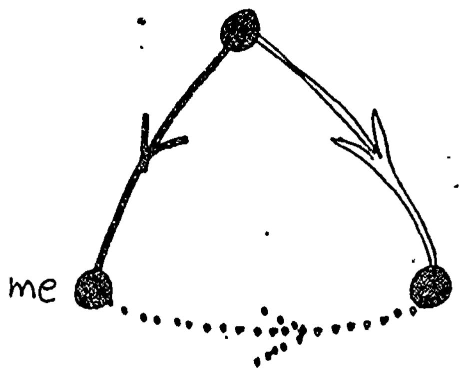

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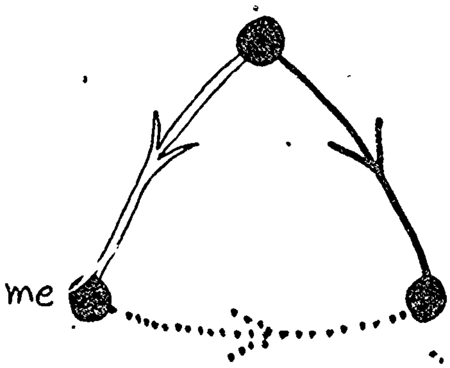

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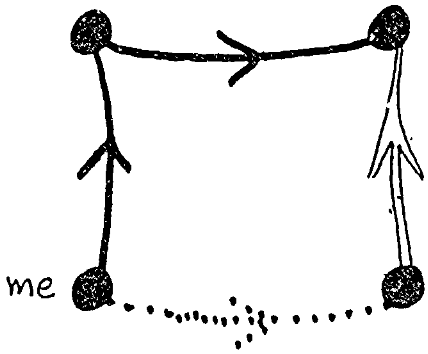
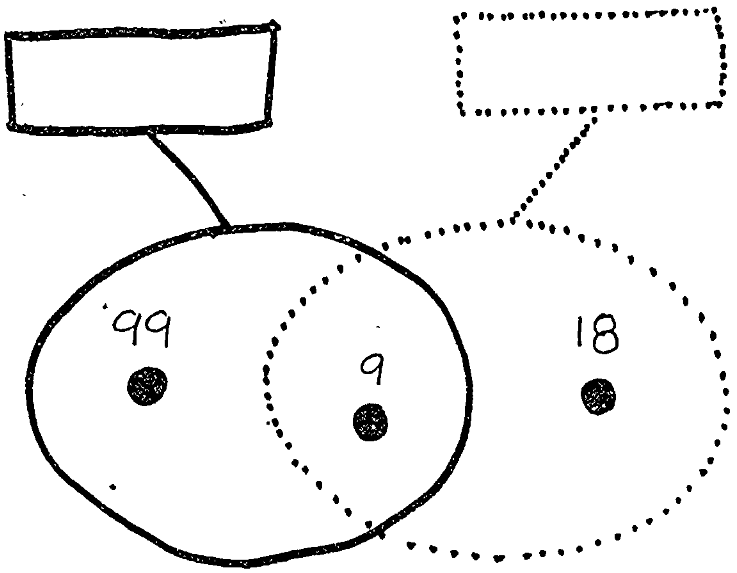


_ _ _ _ _ _ _ _ _ _ _

Name ______________________ L 24 ***

Cross out the labels that the strings can not have.



SOLID	DOTTED
multiples of 2	multiples of 2
multiples of 3	multiples of 3
multiples of 4	multiples of 4
multiples of 5	multiples of 5
multiples of 10	multiples of 10
odd numbers	odd numbers
positive prime numbers	positive prime numbers
larger than 50	larger than 50
smaller than 50	smaller than 50
larger than -10	larger than -10
smaller than -10	smaller than -10
positive divisors of 12	positive divisors of 12
positive divisors of 18	positive divisors of 18
positive divisors of 20	positive divisors of 20
positive divisors of 24	positive divisors of 24
positive divisors of 27	positive divisors of 27

The label for the solid string is $\boxed{}$

the label for the dotted string could be

[ ] or [ ]

It is your turn in the String Game.
You want to find the dotted string.

1) You can find the label for the dotted string by playing exactly one of these numbers, even if you get a **NO** answer. Circle the number you should play.

3; 105; 60; 7; 2.

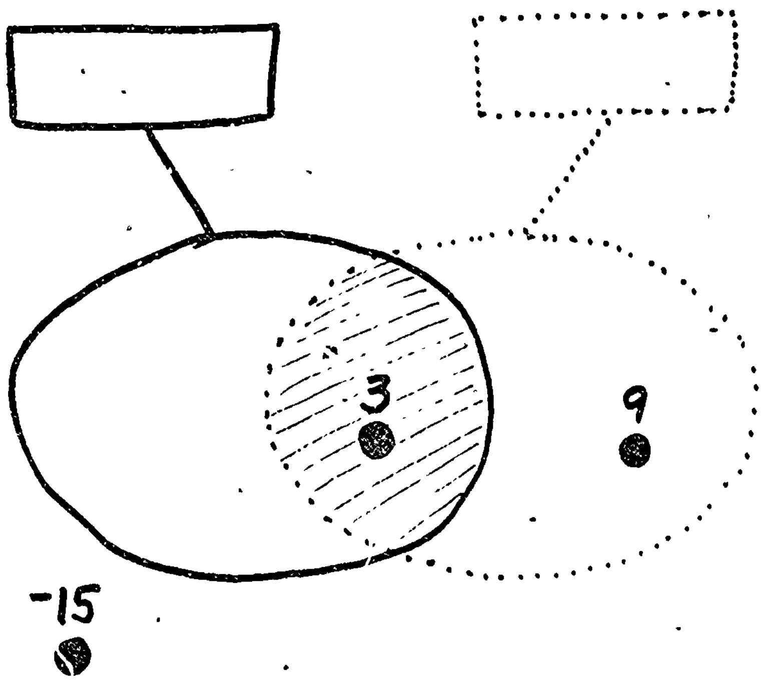
2) Repeat problem (1) but with these numbers.

20; 100; 6; 55; 1.

Name ______________________________ L 28 ***

Cross out the labels that the strings cannot have.
The hatching is also a clue.

Complete the tags.



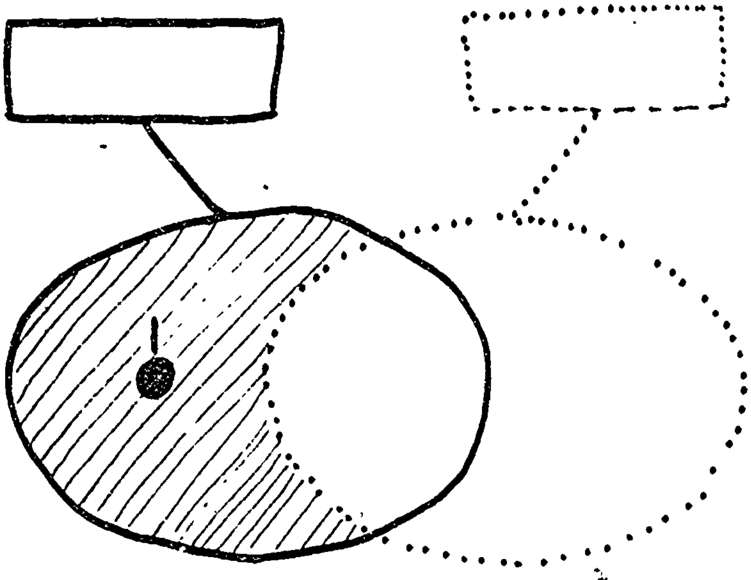
SOLID	DOTTED
multiples of 2	multiples of 2
multiples of 3	multiples of 3.
multiples of 4	multiples of 4
multiples of 5	multiples of 5
multiples of 10	multiples of 10
odd numbers	odd numbers
positive prime numbers	positive prime numbers
larger than 50	larger than 50
smaller than 50	smaller than 50
larger than -10	larger than -10
smaller than -10	smaller than -10
positive divisors of 12	positive divisors of 12
positive divisors of 18	positive divisors of 18
positive divisors of 20	positive divisors of 20
positive divisors of 24	positive divisors of 24
positive divisors of 27	positive divisors of 27

Name ______________________ L 29 ****

Cross out the labels that the strings cannot have.

The hatching is also a clue.

Complete the tags.



SOLID	DOTTED
multiples of 2	multiples of 2
multiples of 3	multiples of 3
multiples of 4	multiples of 4
multiples of 5	multiples of 5
multiples of 10	multiples of 10
odd numbers	odd numbers
positive prime numbers	positive prime numbers
larger than 50	larger than 50
smaller than 50	smaller than 50
larger than -10	larger than -10
smaller than -10	smaller than -10
positive divisors of 12	positive divisors of 12
positive divisors of 18	positive divisors of 18
positive divisors of 20	positive divisors of 20
positive divisors of 24	positive divisors of 24
positive divisors of 27	positive divisors of 27

So much for my examples of the mathematician's contribution to curriculum development. I make no claim that I have surveyed the field. In fact, cutting back has been a major problem. (How I could develop such a talk without a single example from Arthur Engel is almost beyond comprehension.) But providing a survey was not my intent; my intent was rather to provide a setting for the conclusions to which I now turn.

Has the Mathematician Anything to Offer?

First and foremost let me state unequivocally that my answer to that titular question is YES! The mathematician has in fact much to offer. And in a tradition that extends back hundreds of years mathematicians have indeed already contributed to curriculum development. I suggest that a nice starting point for a historical analysis of such contributions might well be Hopital's the Bernoullis' long succession of textbooks on the calculus which first exposed this esoteric study of a few specialists to a wide range of scientists.

It is interesting to note in passing that the Bernoullis were severely criticized in the literature of the day for their contribution: shades of New Math! We should recall in this regard that mathematics was first something to be kept secret and exposed only to selected students and followers. Scholars from the time of Pythagoras down to that of Tartaglia made it their business to guard their knowledge well. We are the unfortunate inheritors of that history - no longer by volition, today more by inattention.

In fact it is this very tradition of the mathematician as high priest, the mathematician as magician, the mathematician as guru that creates a major problem with their playing their appropriate role. We have always dealt with mathematicians in an intellectually unsatisfying way. Instead New Math essentially took the form of the mathematician saying to the classroom teacher, "Here it is; take it or leave it." There was no dialog. No one ever said to Ed Moise, "That is just plain stupid," about a particular section in his program. As a matter of fact evaluations by teachers using SMSG were without exception highly favorable. One possible reason: They were paid to submit the evaluations and didn't want to kill the goose.

I do not place the blame for this lack of dialog all on the mathematician although he certainly deserves a full share. There was (is?) no one available to speak up, to question, to test, to argue, to share in the development. The parent-child relationship that developed and continues today is not a setting for the kind of adult to adult dialog that was and is needed. That transactional analysis model is a useful one here because it poses the problem we face in terms that may be simplistic but that still get to the heart of the matter.

Unfortunately we have great difficulty in responding to this situation. There just are not enough highly talented classroom teachers around who can and will work with mathematicians - if indeed we can ever again free up a mathematician or two to work on curriculum development. And there is no middle group of math educators to play this needed role. That, of course, is the real tragedy, the deflection of the concerns of the math educator into the silliest, most trivial kind of research at a time when we need curricular research and development in the worst way. Now our U. S. governmental agencies are exacerbating this situation by putting their money on such trivia and fads, withdrawing it from curriculum development. (Just one example of the faddish approach to research: Today's in thing is the work of the Russians. While I welcome the relaxation of tight controls in the so-called Russian teaching experiment, I find their intellectual analyses often mindless and usually Lysenko-like tied to political dogma. But we are dropping hundreds of thousands of dollars into this well.)

That's the bad news: placed first because it represents our most serious problem in math education today. Now let me examine my emphatic "yes" closer in order to see if we cannot identify how to make better use of the mathematician's contribution.

I believe that the mathematician has the following things to contribute to curriculum development:

1. He (and of course I am using the pronoun generically here) develops the mathematics.
2. As a more advanced scholar he can often identify prerequisites to that scholarship. I have to admit that I offer this only tentatively and without much faith, that faith having been largely destroyed in the context of the NCTM-MAA negotiations over the achievement decline statement.
3. Speaking from some distance the mathematician tends to take the lid off. He is not always constrained by the classroom teacher's refrain, "Not with these kids, you couldn't." Here I depart from so many of my colleagues in my belief that the Cambridge Conference Goals was a rich and important contribution to curriculum development. I believe that the conference participants were wise to approach their task as what they believed could be done given superior and trained teachers, given appropriate administrative support, and given schools in control, even though those resources are not at hand. Why, by the way, with all the money we spend on education do we not set up demonstration centers where we could attempt to reach or even breach these limits?
4. The good mathematician - so rare! - is comfortable with the concepts with which he works and can often identify means of communication from this vantage point. This comment is of course a two-edged sword. It says something about the mathematicians who write those sterile definition-theorem-proof compendiums that pose as texts.

This says more than don't write what you don't know. Writing well about ideas requires cohabitation with those ideas: knowing them, befriending them, using them roughly. It is this kind of intimate day-to-day knowledge that the working mathematician has to offer.

5. The mathematician takes problem solving -- as opposed here to teaching algorithms --seriously.

6. An extension of this is the mathematician's ability to tolerate lack of closure. Ambiguity is the bane of the existence of the low level math teacher. In fact one of the quite negative attractions of mathematics, I think, is its seeming closure and completeness. This draws to mathematics teaching people who deal only at the right-wrong, yes-no, black-white dichotomy level. And we wonder why so much math in their hands degenerates to the "repeat back to me in my words" level of exposition. The mathematician <u>can</u> and <u>should</u> offer a counterbalance to this. (I hope that they will address this math in the courses subgroup.)

7. Partly because he has little political investment, the mathematician can supply courage. He can imagine the impossible. Don't discharge that as merely quixotic. Surely that is exactly how mathematics itself advanced. When everyone else is prepared to reject subtracting eight from five, the mathematician barges ahead. Surely there is a contribution here to our thinking about curriculum.

8. An aspect of this is the mathematician's ability to see the forest through the trees. He can often at the same time perceive <u>both</u> local <u>and</u> global. The creative mathematician, it seems to me, has both brain hemispheres actively functioning in reinforcing rather than inhibiting ways. It is this ability, I suspect, that lets the mathematician see the alternate path to the destination, to see the dead ends,

and to see how apparent dead ends actually require only minor re-direction to become productive avenues.

At my own university we had a recent example of these points. An outside committee was brought in to investigate an intolerable situation in our Faculty of Natural Sciences and Mathematics. On that committee sat Stanislav Ulam, Herbert Robbins and other senior scientists. They quickly cut through to the internal organs and exposed the nerves. I see a perfect parallel here with the mathematician's approach to curriculum development.

And finally 9. Despite a spotty record, the mathematician has made important contributions. I hope that my examples have shown that.

Some of you may not feel that these points are important enough to include the mathematician in. To you I suggest you consider the alternative: Is it right to exclude these experts? I think not and I think that it is the rare curriculum development project that excludes them - including the Ontario programs that have done so - that doesn't suffer from this exclusion.

To my colleagues in mathematics education I say: what we need to do is to find the means to work with a few good mathematicians on curriculum development. But we need even more to stiffen our own backs so that the relationship is good. And we need to identify and support more non-mathematicians like Wirtz in settings where they can interact with mathematicians as equals.

So now let's suppose we have established the working relationship. What are some of the things we must watch out for? I list a few:

1. Many mathematicians have their own blind spots, sometimes their own specialty. Hilton: We can develop category theory for grade eight. Sometimes techniques. Zachnias: Successive approximations will solve everything. Sometimes a focus on trivia. Sometimes admitting only exactly what has been done before. Sometimes a total focus on the negative, the Waterloo posture. All this says is that mathematicians are exactly like the rest of us and should be treated as such.
2. Deference should be given to mathematicians on content issues only, and never on pedagogical issues. Witness in this regard the general quality of college instruction.
3. Mathematicians love bandwagons just like so many of the rest of us. Witness Carl Allendorfer's adoption of behavioral objectives, Ed Begle's turn to educational research for answers - "it has been established that homework has no affect on learning mathematics" - , the capitulation of Paul Rosenbloom to the scientists, the ready turnover of curriculum development to the behavioral psychologists.
4. Curriculum development represents a professional sacrifice to young university mathematicians and should be viewed by us in this light. When we find them uncomfortable we should recognize their discomfort as real and reasoned. We should in fact seek ways of responding to this discomfort.
5. Most important we must seek ways to free the mathematicians from their own hangups, for example their journal-mode of response to any question. We need to encourage them to probe the way their themselves

think, to force them in effect to be honest to themselves as well as to us.

A couple of you have commented negatively about this. Who cares how the mathematician thinks? What I am saying here is an attempted response to this. How he thinks is, I believe, what mathematics is about. We can Pooh to students more for mathematical pathologies than for mathematical thinking. The student thinking analysis takes mathematics instruction in its current mode as right and proper - but I have argued here that that mode is not necessarily right and proper. The mathematician's mode of thought - if we can lever it out of him - may help give us direction.

By these means and others we can establish the dialog at an appropriate level between mathematician and curriculum developer.